

NOTES #1 Arithmetic Sequence

Sequence

- an ordered list of numbers

Arithmetic Sequence

- a sequence of values that follows a **pattern of adding a fixed amount** from one term to the next. The number added to each term is constant.

Common Difference

- The **fixed amount added** is called the **common difference, d** . To find the common difference, subtract the first term from the second term.

$$(d = 2^{\text{nd}} \text{ term} - 1^{\text{st}} \text{ term})$$

Example 1: Is the given sequence arithmetic? If so, identify the common difference.

A. 2, 5, 7, 12, ...

$2 \rightarrow 5$
 $5 \rightarrow 7$
 $7 \rightarrow 12$

not arithmetic

B. 48, 45, 42, 39, ...

$48 \rightarrow 45$
 $45 \rightarrow 42$
 $42 \rightarrow 39$

arithmetic

$$d = -3$$

Example 2: Find the common difference of each sequence. Then find the next term.

A. 10, 4, -2, -8, ...

$10 \rightarrow 4$
 $4 \rightarrow -2$
 $-2 \rightarrow -8$

$$d = -6$$

next term -14

B. $4, x+4, 2x+4, 3x+4, \dots$

$4 \rightarrow x+4$
 $x+4 \rightarrow 2x+4$
 $2x+4 \rightarrow 3x+4$

$$d = x$$

$$4x+4$$

Arithmetic Sequence Formulas

Recursive Formula

$$a_1 = \text{a given value}$$

$$a_n = a_{n-1} + d$$

Next = Now + Common Difference

Explicit Formula

$$a_n = a_1 + (n-1)d$$

In these formulas,

a_n is the n th term

a_1 is the first term

n is the number of the term

d is the common difference

You can think of each term in the sequence as:

$$a_1, a_2, a_3, a_4, \dots, a_{n-1}, a_n, \dots$$

↑ ↑ ↑
Now next

Example 3: Write a recursive formula and an explicit formula for each sequence.

$d=3$ A. 2, 5, 8, ...

R $a_1 = 2$

$a_n = a_{n-1} + 3$

$a_n = 2 + (n-1)3$

B. -32, -20, -8, 4, 16, ...

$d=12$

$a_1 = 32$

$a_n = -32 + (n-1)12$

Example 4: Tell if the formula is RECURSIVE or EXPLICIT. Find the first 5 terms.

Recursive

A. $a_n = a_{n-1} + 5$

3, 8, 11, 14, 17
+3

Explicit

B. $a_n = 3n - 5$

$a_1 = 3 \cdot 1 - 5 = -2$

$a_{100} = 3 \cdot 100 - 5 = 295$

Example 5: Write the 1st six terms of the arithmetic sequence in which $a_1 = 5$ and $d = \frac{1}{2}$.

5, 5.5, 6, 6.5, 7

Example 6: Find the 17th term of the following sequence: 23, 30, 37, 44, ...

a_1 $d=7$

$a_n = 23 + (n-1)7$

$a_{17} = 23 + (17-1)7 = 135$

$a_n = 16 + 7n$ $a_{17} = 16 + 7 \cdot 17 = 135$

ARITHMETIC MEAN: Sum of two values divided by 2 $\left(\frac{j+k}{2}\right)$

Example 7: Given two terms of each arithmetic sequence, find a_7 and d .

A. $a_2 = 10$ and $a_4 = 18$

$\frac{10}{a_2} \quad \frac{18}{a_4}$

$a_3 = \frac{(10+18)}{2} = 14$

$d=4$

$a_1 = 6$

B. $a_3 = 55$ and $a_7 = 11$

$\frac{55}{a_3} \quad \frac{11}{a_7}$

$a_5 = \frac{(55+11)}{2}$

$a_5 = 33$

$a_4 = \frac{(55+33)}{2} = 44$

$d = -11$
 $a_1 = 77$