

AP Statistics  
Chapter 6 Practice Test

Name Key  
Date \_\_\_\_\_ Hour \_\_\_\_\_

Questions T6.1 and T6.2 refer to the following setting. A psychologist studied the number of puzzles that subjects were able to solve in a five-minute period while listening to soothing music. Let  $X$  be the number of puzzles completed successfully by a subject. The psychologist found that  $X$  had the following probability distribution:

Value of $X$ :	1	2	3	4
Probability:	0.2	0.4	0.3	0.1

T6.1. What is the probability that a randomly chosen subject completes at least 3 puzzles in the five-minute period while listening to soothing music?

$$P(X \geq 3) = P(X=3) + P(X=4) \\ = 0.3 + 0.1 = 0.4$$

- (a) 0.3 (b) 0.4 (c) 0.6 (d) 0.9 (e) Cannot be determined

T6.2. Suppose that three randomly selected subjects solve puzzles for five minutes each. The expected value of the total number of puzzles solved by the three subjects is

$$\mu_X = E(X) = 1(0.2) + 2(0.4) + 3(0.3) + 4(0.1) = 2.3$$

The expected value for 3 subjects is  $3 \cdot \mu_X = 3(2.3) = 6.9$

- (a) 1.8 (b) 2.3 (c) 2.5 (d) 6.9 (e) 7.5

T6.3. Suppose a student is randomly selected from your school. Which of the following pairs of random variables are most likely independent?

- (a)  $X$  = student's height;  $Y$  = student's weight *most likely related*  
 (b)  $X$  = student's IQ;  $Y$  = student's GPA *most likely related*  
 (c)  $X$  = student's PSAT Math score;  $Y$  = student's PSAT Verbal score *most likely related*  
 (d)  $X$  = average amount of homework the student does per night;  $Y$  = student's GPA *most likely related*  
 (e)  $X$  = average amount of homework the student does per night;  $Y$  = student's height

T6.4. A certain vending machine offers 20-ounce bottles of soda for \$1.50. The number of bottles  $X$  bought from the machine on any day is a random variable with mean 50 and standard deviation 15. Let the random variable  $Y$  equal the total revenue from this machine on a given day. Assume that the machine works properly and that no sodas are stolen from the machine. What are the mean and standard deviation of  $Y$ ?

- (a)  $\mu_Y = \$1.50$ ,  $\sigma_Y = \$22.50$   
 (b)  $\mu_Y = \$1.50$ ,  $\sigma_Y = \$33.75$   
 (c)  $\mu_Y = \$75$ ,  $\sigma_Y = \$18.37$   
 (d)  $\mu_Y = \$75$ ,  $\sigma_Y = \$22.50$

$X$  = # of bottles bought from machine on any given day  
 $Y$  = total revenue from machine any given day  
 $Y = \$1.50X$   
 $\mu_X = 50$   $\sigma_X = 15$   
 $\mu_Y = 50(\$1.50) = \$75$   $\sigma_Y = 15(\$1.50) = \$22.50$

(e)  $\mu_Y = \$75$ ,  $\sigma_Y = \$33.75$

Questions T6.5 and T6.6 refer to the following setting. The weight of tomatoes chosen at random from a bin at the farmer's market is a random variable with mean  $m = 10$  ounces and standard deviation  $s = 1$  ounce. Suppose we pick four tomatoes at random from the bin and find their total weight  $T$ .

T6.5. The random variable  $T$  has a mean of

- (a) 2.5 ounces (b) 4 ounces (c) 10 ounces (d) 40 ounces (e) 41 ounces

$x =$  weight of tomato  $\mu_x = 10 \text{ oz}$   $\sigma_x = 1 \text{ oz}$

$T =$  total weight of 4 tomatoes

$T = x_1 + x_2 + x_3 + x_4$

So  $\mu_T = \mu_{x_1} + \mu_{x_2} + \mu_{x_3} + \mu_{x_4} = 10 + 10 + 10 + 10 = 40 \text{ oz}$

T6.6. The random variable  $T$  has a standard deviation (in ounces) of

- (a) 0.25 (b) 0.50 (c) 0.71 (d) 2 (e) 4

$\sigma_T = \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2 + \sigma_{x_4}^2} = \sqrt{(1)^2 + (1)^2 + (1)^2 + (1)^2} = \sqrt{4} = 2 \text{ oz}$

T6.7. Which of the following random variables is geometric?

- (a) The number of times I have to roll a die to get two 6s. *not a 1st success*  
(b) The number of cards I deal from a well-shuffled deck of 52 cards until I get a heart. *not replacing, not independent*  
(c) The number of digits I read in a randomly selected row of the random digits table until I find a 7. BITS ✓  
(d) The number of 7s in a row of 40 random digits.  $n = 40$   
(e) The number of 6s I get if I roll a die 10 times.  $n = 10$

T6.8. Seventeen people have been exposed to a particular disease. Each one independently has a 40% chance of contracting the disease. A hospital has the capacity to handle 10 cases of the disease. What is the probability that the hospital's capacity will be exceeded?  $P(X > 10)$

- (a) 0.011 (b) 0.035 (c) 0.092 (d) 0.965 (e) 0.989

Binomial Setting  $B(17, .40)$

B ✓ s = has disease f = does not have disease

I ✓ yes

N ✓ yes  $n = 17$

S ✓ yes  $p = .40$  is same for all trials

$$P(X > 10) = 1 - P(X \leq 10)$$

$$0.1 \dots 10 / (11 \dots 17) = 1 - \text{binomcdf}(17, .40, 10) \approx .0348$$

where  $n=17, p=.40, x=10$

T6.10. A test for extrasensory perception (ESP) involves asking a person to tell which of 5 shapes—a circle, star, triangle, diamond, or heart—appears on a hidden computer screen. On each trial, the computer is equally likely to select any of the 5 shapes. Suppose researchers are testing a person who does not have ESP and so is just guessing on each trial. What is the probability that the person guesses the first 4 shapes incorrectly but gets the fifth correct?

- (a)  $1/5$

- (b)  $(\frac{4}{5})^4$

- (c)  $(\frac{4}{5})^4 \cdot (\frac{1}{5})$

- (d)  $(\frac{5}{1}) (\frac{4}{5})^4 \cdot (\frac{1}{5})$

- (e)  $4/5$

Geometric

4 incorrect

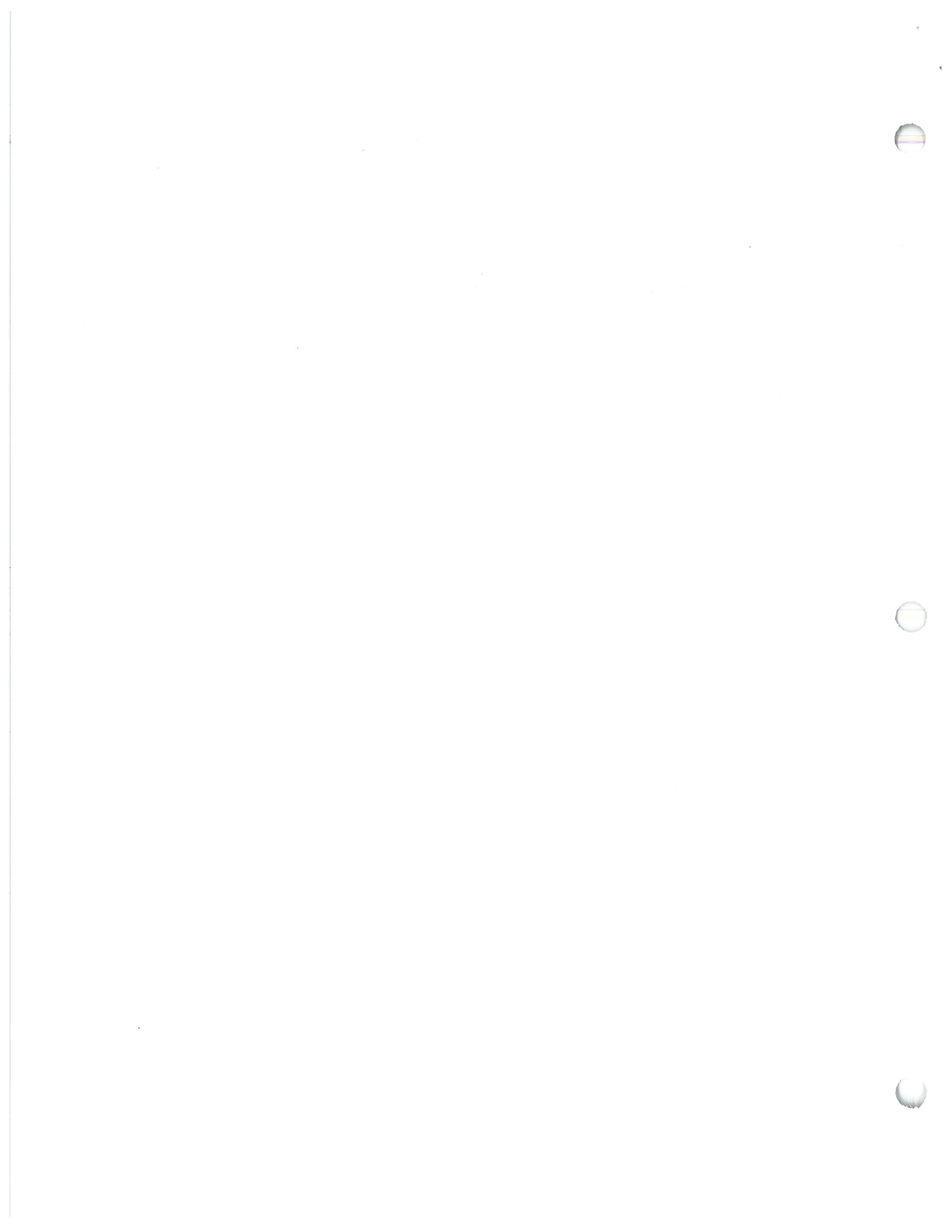
1 correct

$$(\frac{4}{5})^4 \cdot (\frac{1}{5})$$

$p = 1/5$  (correct)

$1-p = 4/5$  incorrect

$y = \#$  of guesses to get 1 correct





AP STATS  
FREE RESPONSE PRACTICE TEST

Name Key  
Date \_\_\_\_\_ Hour \_\_\_\_\_

Section II: Free Response Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

T6.11. Let  $Y$  denote the number of broken eggs in a randomly selected carton of one dozen "store brand" eggs at a local supermarket. Suppose that the probability distribution of  $Y$  is as follows.

Value $y_i$ :	0	1	2	3	4
Probability $p_i$ :	0.78	0.11	0.07	0.03	0.01

(a) What is the probability that at least 10 eggs in a randomly selected carton are unbroken? <sup>so 0, 1, or 2 are broken</sup>

$Y = \#$  of broken eggs

$$P(Y \leq 2) = .78 + .11 + .07 = .96$$

(b) Calculate and interpret  $\mu_Y$ . SHOW YOUR WORK!!!

$$\mu_Y = E(Y) = 0(.78) + 1(.11) + 2(.07) + 3(.03) + 4(.01) = .38 \text{ eggs}$$

If we inspected many, many cartons of eggs, we would expect to see about .38 broken eggs, on average.

(c) Calculate and interpret  $\sigma_Y$ . SHOW YOUR WORK!!!!

$$\sigma_Y = \sqrt{(0-.38)^2(.78) + (1-.38)^2(.11) + (2-.38)^2(.07) + (3-.38)^2(.03) + (4-.38)^2(.01)} \approx .822 \text{ eggs}$$

If we selected a random carton of eggs, we would expect the # of broken eggs to vary from the mean of .38 eggs by .822 eggs, on average.

(d) A quality control inspector at the store keeps looking at randomly selected cartons of eggs until he finds one with at least 2 broken eggs. Find the probability that this happens in one of the first three cartons he inspects.

$Z = \#$  of cartons inspected until he finds at least 2 broken eggs (until  $Y \geq 2$ )

Geometric

success is  $Y \geq 2$

$$P(Y \geq 2) = .07 + .03 + .01 = .11 \leftarrow p$$

2 or more broken eggs      so  $1-p = .89$

$$\begin{aligned} \text{So } P(Z \leq 3) &= P(Z=1) + P(Z=2) + P(Z=3) \\ &= \frac{1}{15} + \frac{1}{15} + \frac{2}{15} \\ &= .11 + .11(.89) + .11(.89)^2 \approx .295 \\ \text{or } P(Z \leq 3) &= \text{geomcdf}(.11, 3) \approx .295 \\ &\text{where } a = .11 \text{ and } b = 2 \end{aligned}$$

T6.12. *Ladies Home Journal* magazine reported that 66% of all dog owners greet their dog before greeting their spouse or children when they return home at the end of the workday. Assume that this claim is true. Suppose 12 dog owners are selected at random. Let  $X$  = the number of owners who greet their dogs first.

(a) Explain why  $X$  is a binomial random variable.

B: yes  $s$  = greet dog 1st  $f$  = not greet dog 1st

I: yes, dog owners selected at random so behaviors should be independent

N: yes,  $n = 12$

S: yes,  $p = .66$  is the same for all trials

$\therefore$  binomial random variable  $B(12, .66)$

(b) Only 4 of the owners in the random sample greeted their dogs first. Does this give convincing evidence against the *Ladies Home Journal* claim? Calculate an appropriate probability to support your answer.

$$P(X=4) = \text{binompdf}(12, .66, 4) \approx .017 \approx 1.7\%$$

where  $n=12$ ,  $p=.66$  and  $x=4$

$$\mu_x = 12(.66) \approx 7.92 \text{ owners}$$

4 is below expected, so

small chance

also  $\checkmark P(X \leq 4)$

$$P(X \leq 4) = \text{binomcdf}(12, .66, 4) \approx .0213 \approx 2.13\%$$

where  $n=12$ ,  $p=.66$  and  $x=4$

still very small chance

for 4 owners or less

The probability that 4 owners or less will greet their dog 1st is very small, about 2%, this result is reasonably unlikely. This does provide evidence that the claim of 66% is incorrect.

T6.13. Ed and Adelaide attend the same high school but are in different math classes. The time  $E$  that it takes Ed to do his math homework follows a Normal distribution with mean 25 minutes and standard deviation 5 minutes. Adelaide's math homework time  $a$  follows a Normal distribution with mean 50 minutes and standard deviation 10 minutes.

- (a) Randomly select one math assignment of Ed's and one math assignment of Adelaide's. Let the random variable  $D$  be the difference in the amount of time each student spent on their assignments:  $D = a - E$ . Find the mean and the standard deviation of  $D$ . Show your work.

$$E = \text{time Ed takes to do math Hw} \quad a = \text{time Adelaide takes to do math Hw}$$

$$N(25, 5) \quad N(50, 10)$$

$D$  = difference in the amount of time each student spent on assignments

$$D = a - E$$

$$\mu_D = \mu_a - \mu_E = 50 - 25 = 25 \text{ min}$$

$$\sigma_D = \sqrt{\sigma_a^2 + \sigma_E^2} = \sqrt{(5)^2 + (10)^2} = \sqrt{125} \approx 11.18 \text{ min}$$

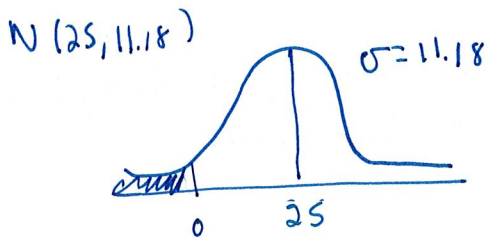
(assuming amt of time they spend on Hw is independent of each other.)

So  $D$  would be  $N(25, 11.18)$

- (b) Find the probability that Ed spent longer on his assignment than Adelaide did on hers. Show your work.

If Ed spends more time on his Hw, that means  $E > A \Rightarrow D = A - E < 0$

$$P(D < 0) = \text{normcdf}(-10, 0, 0, 25, 11.18) \approx \underline{\underline{.0127}}$$



There is about a 1.27% chance that Ed takes longer than Adelaide on a randomly chosen assignment.



T6.14. According to the Census Bureau, 13% of American adults (aged 18 and over) are Hispanic. An opinion poll plans to contact an SRS of 1200 adults.

(a) What is the mean number of Hispanics in such samples? What is the standard deviation?

$X = \#$  of Hispanics in the sample

This is a Binomial Setting:

- B:  $\checkmark$  s: Hispanic f: Not Hispanic
- I:  $\checkmark$  randomly selected so opinions are independent
- N:  $\checkmark$   $n = 1200$
- S:  $\checkmark$   $p = .13$  is the same for every trial

so  $\mu_X = np = 1200(.13) = 156$  Hispanics

$\sigma_X = \sqrt{np(1-p)} = \sqrt{1200(.13)(.87)} \approx 11.65$  Hispanics

(b) Should we be suspicious if the sample selected for the opinion poll contains 15% Hispanic people? Compute an appropriate probability to support your answer.

If sample contains 15% Hispanics, that means there were 180 Hispanics.  
 $(1200(.15) = 180)$

$P(X=180) = \text{binompdf}(1200, .13, 180) \approx .004 \approx .4\%$   
where  $n=1200$ ,  $p=.13$ , and  $x=180$

Since 180 is more than  $\mu_X$ , let's look at prob of at least 180 Hispanics

$P(X \geq 180) = 1 - \text{prob}(X \leq 179) = 1 - \text{binomcdf}(1200, .13, 179) \approx .023 \approx 2.3\%$   
where  $n=1200$ ,  $p=.13$ , and  $x=179$   
 $0, \dots, 179, (180, 181, \dots, 1200)$

The probability of having 180 or more Hispanics is very small, about 2.3%, which means it would be very unlikely to happen by chance. Therefore, we should be a little suspicious if this were to occur.