Name: Key Hour: _____ Date: _____

Learning Targets

- Construct and interpret a confidence interval for a population proportion.
- Determine the sample size required to obtain a C% confidence interval for a population proportion with a specified margin of error

Lesson 8.2: Day 2: How much of the Earth is covered by water?



What proportion of the Earth is covered by water? We will investigate this question by taking a random sample of locations on the globe.

1. How many locations did your class sample? ______ How many locations were water? ______
2. Calculate the proportion of locations from your sample that are water. $\hat{p}=$ ______
3. Construct a 95% confidence interval to estimate the proportion of the Earth that is water.

STATE: State the parameter you want to estimate and the confidence level.

Estimate the

Parameter: true proportion of Earth covered by water at a Confidence level: 95%

PLAN: Identify the appropriate inference method and check conditions.

Name of procedure: One sample z interval for p.

Check conditions:

① Random: We took a random sample of locations.

② 10%: $10(n) <$ all locations on globe

③ Large Counts
$n(\hat{p}) \geq 10$
$n(1-\hat{p}) \geq 10$

DO: If the conditions are met, perform the calculations.

General Formula for any confidence interval: Point Estimate ± Margin of Error

Specific Formula for this confidence interval: $\hat{p} \pm z^{*}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

This is the one-sample z interval for population proportion (p)

Plug numbers into the formula:

Answer:

CONCLUDE: Interpret your interval in the context of the problem.

Interpret: We are 95% confident that the interval from _____ to _____ captures the true proportion of Earth that is covered by water.

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Lesson 8.2 Day 2 – The Four Step Process

Important ideas:

L.T. #1 The Four Step Process

- State: Parameter and % level
  - Estimate the true proportion p of (context) at a ____ confidence level (Use this sentence!)
- Plan: Name of Procedure and Check Conditions
  Procedure we learned in 8.2 is One Sample z-interval for p
  $$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
- Do: General + Specific Formulas
  Plug in #'s, final Interval

Conclude: Interpret Interval in Context
"We are ____% confident, . . .

L.T. #2 Choosing Sample Size

$$M.E. \geq z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Solve for n.

* Always round up! (smaller sample size results in larger M.E.)

* If $\hat{p}$ is unknown, use $\hat{p} = 0.5$, that's conservative

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Check Your Understanding

A company has received complaints about its customer service. The managers intend to hire a consultant to carry out a survey of customers. Before contacting the consultant, the company president wants some idea of the sample size that she will be required to pay for. One value of interest is the proportion p of customers who are satisfied with the company's customer service. She decides that she wants the estimate to be within 3 percentage points (0.03) at a 95% confidence level.

M.E. $z^* = \text{invNorm}\left(\frac{1-.95}{2}\right)$

1. Using a conservative estimate for $\hat{p}$, how large of a sample is needed? $\hat{p} = 0.5$

$M.E. \geq z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $\quad z^* = 1.96$

① plug in #'s → $.03 \geq 1.96\sqrt{\frac{(.5)(.5)}{n}}$

② ÷ by 1.96 → $\frac{.03}{1.96} \geq \sqrt{\frac{(.25)}{n}}$

③ Square both sides $\left(\frac{.03}{1.96}\right)^2 \geq \left(\sqrt{\frac{(.25)}{n}}\right)^2$

④ × both sides by n $\left(\frac{.03}{1.96}\right)^2 \geq \frac{.25}{n}$

$n\left(\frac{.03}{1.96}\right)^2 \geq .25$

⑤ ÷ by $(.03/1.96)^2$

$n \geq \frac{.25}{\left(\frac{.03}{1.96}\right)^2}$

$n \geq 1067.11$

$\boxed{n = 1068 \text{ people}}$

2. In the company's prior-year survey, 80% of customers surveyed said they were satisfied. Using this value as a guess for $\hat{p}$, find the sample size needed for a margin of error of at most 3 percentage points with 95% confidence. How does this compare with the required sample size from question #1? $\hat{p} = .80$

$.03 \geq 1.96\sqrt{\frac{.8(.2)}{n}}$

$\frac{.03}{1.96} \geq \sqrt{.16/n}$

Do not round #'s in calculator, be exact

$\left(\frac{.03}{1.96}\right)^2 \geq \frac{.16}{n}$

$n\left(\frac{.03}{1.96}\right)^2 \geq .16$

$n \geq \frac{.16}{\left(\frac{.03}{1.96}\right)^2}$

$n \geq 682.95$

$\boxed{n = 683 \text{ people}}$

sample size decrease from question #1

3. What if the company president demands 99% confidence instead of 95% confidence? Would this require a smaller or larger sample size, assuming everything else remains the same? Explain your answer. $\hat{p} = .80$ ME $= .03$ $z^* = \text{invNorm}\left(\frac{1-.99}{2}\right) = 2.576$

$.03 \geq 2.576\sqrt{\frac{(.8)(.2)}{n}}$

$\frac{.03}{2.576} \geq \sqrt{\frac{.16}{n}}$

$\left(\frac{.03}{2.576}\right)^2 \geq \frac{.16}{n}$

$n \geq \frac{.16}{(.03/2.576)^2}$

$n \geq 1179.7$

$n = 1{,}180$ people

Larger Sample Size

Increasing sample size reduces variability which increases confidence in the interval's ability to capture the parameter.