

Use the following table for questions 1 - 3

The following table compares the hand dominance of 200 Canadian high-school students and what methods they prefer using to communicate with their friends. Suppose one student is chosen randomly from this group of 200.

	Cell phone/Text	In person	Online	Total
Left-handed	12	13	9	34
Right-handed	43	72	51	166
<b>Total</b>	<b>55</b>	<b>85</b>	<b>60</b>	<b>200</b>

- 1) What is the probability that the student chosen prefers to communicate with friends in person?

$$P(\text{In Person}) = \frac{85}{200} = \frac{17}{40} = .425$$

- 2) If you know the person that has been randomly selected is left-handed, what is the probability that they prefer to communicate with friends in person?

$$P(\text{In Person} | \text{Left Handed}) = \frac{13}{34} \approx .382$$

- 3) You select a student from the group at random. Which of the following statements is true about the events "Left-Handed" and "Prefers to communicate with friends in person"?

(a) The events are mutually exclusive and independent.

(b) The events are not mutually exclusive but they are independent.

(c) The events are mutually exclusive, but they are not independent.

(d) The events are not mutually exclusive, nor are they independent.

(e) The events are independent, but we do not have enough information to determine if they are mutually exclusive.

Another way to see if independent

Does  $P(\text{Left}) \cdot P(\text{In Person}) = P(L \cap I, P)$

$$\frac{34}{200} \cdot \frac{85}{200} = \frac{13}{200}$$

so not independent

Not ME, now  $\checkmark$  for Independence

$$P(\text{Left} | \text{In Person}) = P(\text{Left})$$

$$\frac{13}{85} \neq \frac{34}{200} \text{ no } \neq$$

so not independent

- 4) Event A has probability 0.4. Event B has probability 0.5. If A and B are independent, then the probability that both events occur is .2?

$$P(A) = 0.4 \quad \left\{ \begin{array}{l} \text{Since A and B are independent} \\ P(A \cap B) \end{array} \right. \quad P(A) \cdot P(B) = P(A \cap B)$$

$$P(B) = 0.5 \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad 0.4 \cdot 0.5 = .2$$

- 5) Event A has probability 0.4. Event B has probability 0.5. If A and B are independent, then the probability that either events occur is 0.7?

$$P(A) = 0.4 \quad P(A \text{ or } B)$$

$$P(B) = 0.5$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4 + 0.5 - .2 = 0.7$$

$$P(A \cap B) = (0.4)(0.5) = .2$$

Use the following situation for questions 6 and 7.

Ignoring twins and other multiple births, assume that babies born at a hospital are independent random events with the probability that a baby is a boy and the probability that a baby is a girl both equal to 0.5.

$$P(\text{boy}) = .5 \quad P(\text{girl}) = .5$$

1 time                      1 time

6) What is the probability that the next five babies are girls?

$$P(\text{next 5 babies girls}) = (.5)^5 = \underline{\underline{.03125}}$$

7) What is the probability that at least one of the next three babies is a boy?

$$P(\text{at least 1 baby of next 3 a boy}) = 1 - .125 = \underline{\underline{.875}}$$

$$\textcircled{1} P(\text{at least 1 boy}) = 1 - P(\text{no boys})$$

$$\textcircled{2} P(\text{no boys}) = .5 \quad \text{Prob (no boys 3 times)} = .5^3 = .125$$

8) In a cookie jar, there are 12 chocolate chip cookies, 5 oatmeal raisin cookies, and 7 macadamia nut cookies. What is the probability that if two cookies were chosen (without replacement), that both cookies were the same type of cookie?

$$P(2 \text{ cookies same type}) = P(2 \text{ C.C.}) + P(2 \text{ O.R.}) + P(2 \text{ M.N.})$$

$$= \frac{12}{24} \cdot \frac{11}{23} + \frac{5}{24} \cdot \frac{4}{23} + \frac{7}{24} \cdot \frac{6}{23}$$

$$= \frac{132}{552} + \frac{20}{552} + \frac{42}{552} = \frac{194}{552} \approx \underline{\underline{.3514}}$$

Use the following for questions 9 and 10.

An event A will occur with probability 0.5. An event B will occur with probability 0.4.

The probability that both A and B will occur is 0.2.

$$P(A) = 0.5$$

$$P(B) = 0.4$$

$$P(A \cap B) = 0.2$$

9) What is the conditional probability of A, given B?

$$P(A|B) = \frac{.2}{.4} = \underline{\underline{.5}}$$

10) We may conclude that

(a) events A and B are independent.

(b) events A and B are mutually exclusive. *no*  $P(A \cap B) \neq 0$

(c) either A or B always occurs. *no*  $P(A) + P(B) \neq 1$

(d) events A and B are complementary. *no*  $P(A) + P(B) \neq 1$

(e) none of the above is correct.

$$\checkmark \text{ by } P(A) \cdot P(B) = P(A \cap B)$$

$$(.5)(.4) = 0.2 \checkmark$$

Independent

$$\text{or } \checkmark \text{ by } P(A|B) = P(A) \text{ or } \checkmark \text{ by } P(B|A) = P(B)$$

$$P(A|B) = .5$$

$$P(A) = .5 \checkmark$$

Independent

$$P(B|A) = \frac{.2}{.5} = .4$$

$$P(B) = 0.4 \checkmark$$

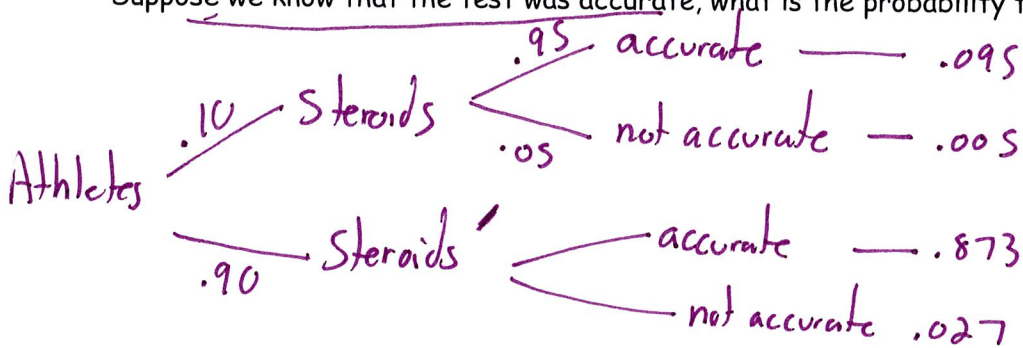
Independent

- 11) Three machines - A, B, and C - are used to produce a large quantity of identical parts at a factory. Machine A produces 60% of the parts, while Machines B and C produce 30% and 10% of the parts respectively. Historical records indicate that 10% of the parts produced by Machine A are defective, compared with 30% for Machine B and 40% for Machine C. What is the probability that a randomly chosen part is defective?

Parts	<u>.60</u>	Machine A	<u>.10</u>	Defective	= .06
	<u>.30</u>	Machine B	<u>.30</u>	Defective	= .09
	<u>.10</u>	Machine C	<u>.40</u>	Defective	= .04

$P(\text{Def}) = P(\text{Def from A}) + P(\text{Def from B}) + P(\text{Def from C}) = .06 + .09 + .04 = \underline{\underline{.19}}$

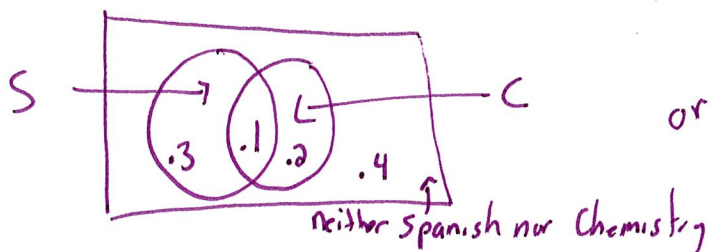
- 12). A company has developed a drug test to detect steroid use by athletes. The test is accurate 95% of the time when an athlete has taken steroids. It is 97% accurate when an athlete hasn't taken steroids. Suppose the drug test will be used in a population of athletes in which 10% have actually taken steroids. Suppose we know that the test was accurate, what is the probability that they didn't take steroids?



$P(\text{Steroids}' | \text{accurate}) = \frac{.873}{(.095 + .873)} = \frac{.873}{.968} = \underline{\underline{.902}}$

- 13) A counselor analyzes student's course selection and calculates the following: The probability that a randomly-chosen student is taking Spanish is 0.4, that the student is taking Chemistry is 0.3, and that the student is taking BOTH Chemistry and Spanish is 0.1.

(a) Let S = Randomly-chosen student is taking Spanish, and C = Randomly-chosen student is taking Chemistry. Sketch a Venn diagram or two-way table that summarizes the probabilities above.



	Spanish	Spanish'	Total
or Chemistry	.1	.2	.3
Chemistry'	.3	.4	.7
Total	.4	.6	1

- (b) Find the probability that a randomly-selected student is taking Spanish OR Chemistry.

$P(S \cup C) = P(S) + P(C) - P(S \cap C) = .4 + .3 - .1 = .6$  or  $P(S \cup C) = .3 + .1 + .2 = .6$

- (c) Find the probability that a randomly-selected student is taking Spanish or isn't taking Chemistry

$P(S \cup C') = P(S) + P(C') - P(S \cap C') = .4 + .7 - .3 = \underline{\underline{.8}}$