

Name: Key Hour: _____ Date: _____

Learning Targets

- State and check the Random, 10%, and Large Counts conditions for performing a significance test about a population proportion.
- Calculate the standardized test statistic and P-value for a test about a population proportion.

Lesson 9.2: Day 1: Are you sure Mrs. Cowells isn't a good free throw shooter?



VS



In Lesson 9.1 we used simulation to estimate a P-value to decide whether or not Mrs. Cowells was exaggerating about her free throw percentage. Today, we will use a formula to find a P-value.

1. We're going to carry out the significance test from lesson 9.1 again. Begin by writing the hypotheses.
- $H_0: p = 0.8$ p : true proportion of free throws Mrs. C makes
 $H_a: p < 0.8$

2. a. Each class found a different P-value because each dotplot was different. Would it be appropriate to use a Normal distribution to model the sampling distribution of \hat{p} ? Justify your answer.
- Since we believe H_0 to be true, we will use p from H_0 when checking Normal condition (large counts)
- $np \geq 10$ so $(.80) \geq 10$ ✓ $n(1-p) \geq 10$ so $(.20) = 10 \geq 10$ ✓

b. Are there any other conditions we should check?

Random and 10% (Independent)

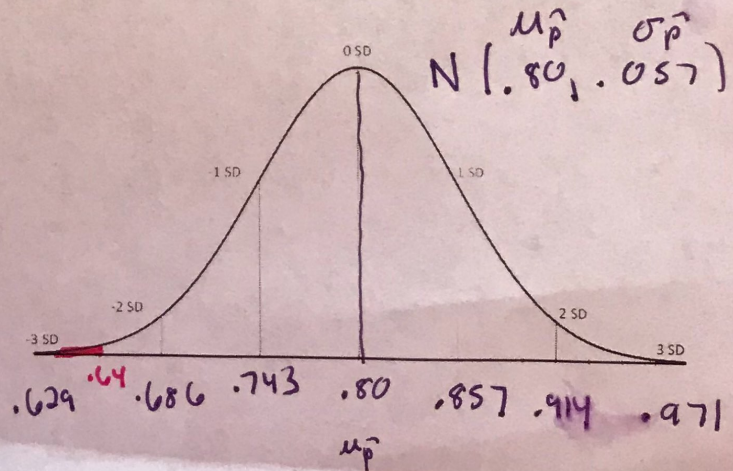
3. Now that conditions have been met, find the mean and standard deviation of the sampling distribution of \hat{p} .

$\mu_{\hat{p}} = p = 0.8$ $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.8(.20)}{50}} = .057$

4. Use the mean and standard deviation you found to label the Normal curve.

5. How many standard deviations below the mean (z-score) is $\hat{p} = 0.64$? Label it on the normal curve.

$z = \frac{.64 - .80}{.057} = -2.81$



6. Find the probability of an 80% shooter making 32/50 ($\hat{p} = 0.64$) or less.

using table A and z-score $P = .0025$

Using Calculator: $P = \text{normcdf}(-1000, .64, .80, .057) = .0025$

7. What conclusion can we make?
 Since $.0025 < .05$, we have convincing evidence against the null hypothesis and reject it. Therefore it appears the true proportion of free throws made by Mrs. C is less than 80%.

Lesson 9.2 Day 1 - Significance Test for p

Important ideas:

T. #1 Significance tests for population proportions Conditions must be met:

1. **Random:** Data should come from a well-designed random sample or random assignment in an experiment. Otherwise we can't infer to the population or establish cause and effect.
2. **Independent:** sampling with replacement for the population allows us to use standard deviation formulas, or if sampling without replacement, we meet the 10% condition for independence $10n > N$
3. **Normal:** sampling distribution of the statistic is approximately normal

** For Hypothesis Tests, we start by assuming the Null H_0 is True, so we will use

p , the population proportion we are assuming to be true from H_0 . ***

(Your textbook will label p from H_0 as p_0 . The AP Formula Sheet just uses p .)
 FOR LARGE COUNTS CONDITION (NORMAL CONDITION) so $p_0 = p$ from H_0

$$np \geq 10 \text{ and } n(1-p) \geq 10$$

T. #2 Calculations: Test Statistic & P-Value

Test Statistic: Measures how far the sample result (\hat{p})
 from the null parameter (p) and in what direction (+/-)
 on a standardized scale. (ex. z-scores)

We use the Test Statistic to find the P-Value

Test Statistic (z-score) =
$$\frac{\text{statistic} - \text{parameter}}{\text{stand. error of the statistic}}$$
 ← ON AP Formula Sheet

$$\text{Test statistic (z)} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$



$N(\mu_{\hat{p}}, \sigma_{\hat{p}})$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

used to calculate → can use table A for z score

P-value

or normal cdf

Always draw picture + label curve $N(\mu_{\hat{p}}, \sigma_{\hat{p}})$

Check Your Understanding

According to the U.S. Census Bureau, the proportion of students in high school who have a part-time job is 0.25. An administrator at a local high school suspects that the proportion of students at her school who have a part-time job is less than the national figure. She would like to carry out a test at the $\alpha = 0.05$ significance level. The administrator selects a random sample of 200 students from the school and finds that 39 of them have a part-time job.

(a) State appropriate hypotheses for performing a significance test. Be sure to define the parameter of interest.

$H_0: p = .25$ $p =$ proportion of high school students who have a part time job.
 $H_a: p < .25$

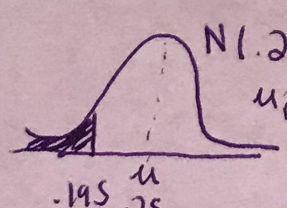
(b) Explain why the sample result gives some evidence for the alternative hypothesis.

$39/200 = .195$ which is less than $.25$

(c) Check if the conditions for performing the significance test are met.

- ① Random: random sample of 200 students ✓
- ② 10% (Independent)
 $10(200) < 2000$ pop of all students at school Reasonable to assume ✓
- ③ Normal: Large counts use p (aka p_0)
 $200(.25) \geq 10 \Rightarrow 50 \geq 10$ ✓
 $200(.75) \geq 10 \Rightarrow 150 \geq 10$ ✓

(d) Calculate the standardized test statistic and P-value.

Test statistic: z-score = $\frac{\text{stat} - \text{parameter}}{\text{st dev of statistic}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.195 - .25}{\sqrt{\frac{.25(.75)}{200}}} = \frac{-.055}{.0306} \approx -1.80$
 $\mu_{\hat{p}} = 0.25$
 $\sigma_{\hat{p}} = .0306$

 $P\text{-value} = \text{Normcdf}(-1.80, .195, .25, .0306) \approx .036$

(e) What conclusion would you make?

Since $\frac{.036}{p} < \frac{.05}{\alpha}$, we have convincing evidence against the null hypothesis and reject it. It appears that the proportion of h.s. students at this school who have part time jobs is less than $.25$.