

Learning Targets

- State and check the Random, 10%, and Large Counts conditions for constructing a confidence interval for a population proportion.
- Determine the critical value for calculating a C% confidence interval for a population proportion using a table or technology.
- Construct and interpret a confidence interval for a population proportion.

Lesson 8.2: Day 1: Which way will the Hershey Kiss land?



When you toss a Hershey Kiss, it sometimes lands flat and sometimes lands on its side. What proportion of tosses will land flat?

Each group of four selects a random sample of 50 Hershey's Kisses to bring back to their desks. Toss the 50 Kisses and then calculate the proportion that land flat. Let \hat{p} = the proportion of the Kisses that land flat.

1. What is your **point estimate** for the true proportion that land flat? $\hat{p} = .42$ (example)

2. Identify the population, parameter, sample and statistic.

Population: All Hershey Kisses Parameter: P = true proportion that land flat
 Sample: 50 Hershey Kisses Statistic: $\hat{p} = .42$

Condition #1

3. Was the sample a random sample? Why is this important?

Yes. It is important so that we can generalize to the

Condition #2

4. What is the formula for calculating the standard deviation of the sampling distribution of \hat{p} ?

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

5. What condition must be met to use this formula? Has it been met?

10% condition: $10(50) <$ population of all Hershey Kisses ✓
 (Independent Condition) Yes it's met
 (must check when sampling w/o replacement)

Condition #3

6. We don't know the value of p (that's the whole point of a confidence interval) so we will use \hat{p} instead. Calculate the standard deviation.

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.42(.58)}{50}} = .0698$$

7. Would it be appropriate to use a normal distribution to model the sampling distribution of \hat{p} ? Justify your answer.

✓ Normal Condition
 Large Counts

$$50(.42) \geq 10 \quad \checkmark$$

$$50(.58) \geq 10 \quad \checkmark$$

Yes it is appropriate.

$$50(.58) \geq 10 \quad \checkmark$$

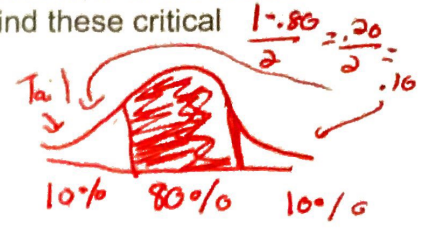
$$29 \geq 10 \quad \checkmark$$

For 80%: using Table A find closest value to .10
 within table = .1003 so $z = -1.28$

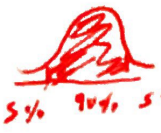
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8. In a normal distribution, 95% of the data lies within 2 standard deviations of the mean. This value is called the critical value. Use table A or InverseNorm to find these critical values:

$invNorm(.10) = -1.282$



80% of the data lies within 1.282 standard deviations of the mean



90% of the data lies within 1.645 standard deviations of the mean

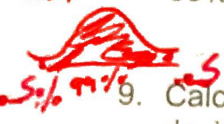
$invNorm(.05) = -1.645$

$invNorm(Tail)$



95% of the data lies within 1.960 standard deviations of the mean

$invNorm(.025) = -1.960$



99% of the data lies within 2.576 standard deviations of the mean

$invNorm(.005) = -2.576$

9. Calculate the margin of error for a 95% interval by multiplying the critical value and standard deviation you found. Show your work.

critical value for 95% = 1.960

$\hat{p} = .0698$

$1.960(.0698) = 0.137$

M.O.E. = critical value \times SE \hat{p}

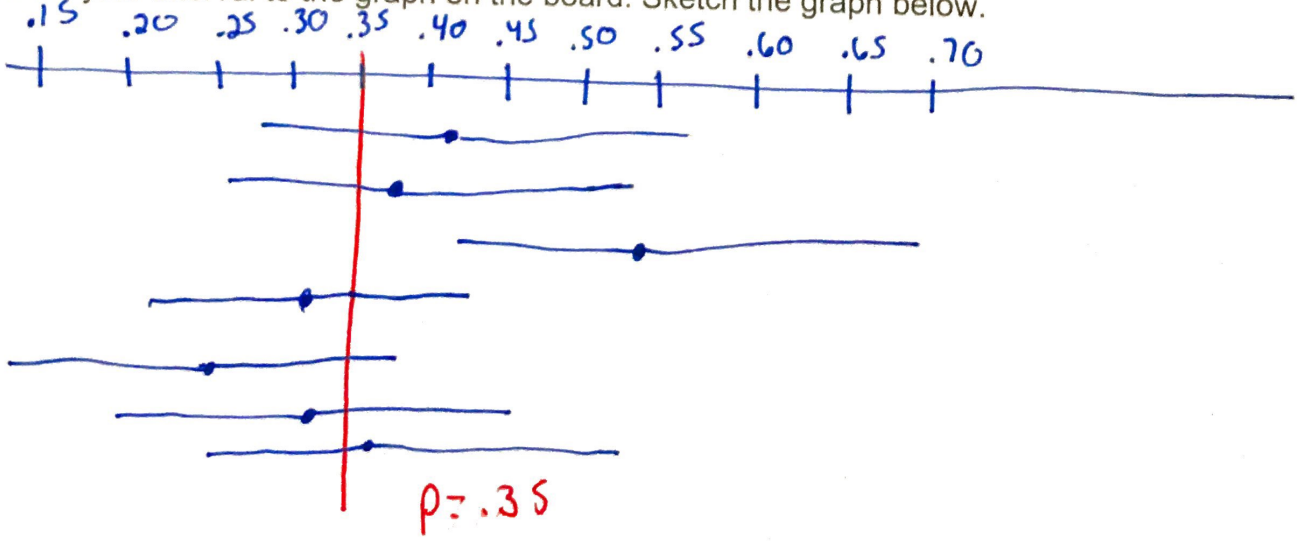
10. Find the 95% confidence interval using point estimate \pm margin of error.

$\hat{p} = .42$
point estimate

$.42 \pm .137 = (.283, .557)$

$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

11. Add your interval to the graph on the board. Sketch the graph below.



12. What do you think is the true proportion of kisses that land flat is?

$p = .35$

Lesson 8.2 Day 1 - Constructing a Confidence Interval for p

Important ideas:

L.T. #1 Conditions Must check 3 conditions when constructing confidence interval

① ✓ Random Condition

② ✓ 10% condition (aka Independent Condition) * must check if sampling w/o replacement **
 $10n < N$

③ ✓ Normal Condition

$$n\hat{p} \geq 10$$

$$n(1-\hat{p}) \geq 10$$

L.T. #2 Critical Values (tells you # of standard deviations statistic (\hat{p}) is within -critical value denoted by " z^* " (z-star)

$$90\%: z^* = 1.645$$

$$95\%: z^* = 1.960$$

$$99\%: z^* = 2.576$$

To find any %, use $\text{invNorm}(\frac{\text{tail}}{2})$
 $\text{invNorm}(\frac{1-\%}{2})$

L.T. #3 Confidence Interval for p

Point Estimate \pm Margin of Error

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Standard Error = SE \hat{p}

** Remember, formula for standard deviation of a sample proportion is
 $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

so when we don't know the true value of p , we replace it with \hat{p} and it is called standard error of a sample proportion **

Check Your Understanding

Sleep Awareness Week begins in the spring with the release of the National Sleep Foundation's annual poll of U.S. sleep habits and ends with the beginning of daylight savings time, when most people lose an hour of sleep. In the foundation's random sample of 1029 U.S. adults, 48% reported that they "often or always" got enough sleep during the last 7 nights.

1. Identify the parameter of interest.

p : true proportion of all U.S. adults who "often or always" got enough sleep during the last 7 nights.

2. Check if the conditions for constructing a confidence interval for p are met.

① Random: a random sample of 1029 U.S. Adults ✓
 ② 10% Condition: $10(1029) < \text{Population all U.S. adults}$ ✓
 (Independent Condition) Reasonable to assume

③ Normal Condition: Large Counts

$$1029(.48) \geq 10$$

$$493.92 \geq 10 \checkmark$$

$$1029(.52) \geq 10$$

$$535.08 \geq 10 \checkmark$$

3. Find the critical value for a 98% confidence interval. Then calculate the interval.


$$\text{Tail Proportion (\%)} = \frac{1-.98}{2} = \frac{.02}{2} = .01$$

$$z^* = \text{invNorm}(.01) = 2.326$$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \rightarrow .48 \pm 2.326 \sqrt{\frac{.48(.52)}{1029}} = .48 \pm .036$$

$$= (.444, .516)$$

4. Interpret the interval in context.

We are 98% confident that the interval from .444 to .516 captures the true proportion of all U.S. adults who report that they "often or always" got enough sleep in the last 7 days.  TheStatsMedic