AP STATS
Chapter 5 Practice Test

Name Key
Date ________________ Hour ______

T5.1. Dr. Stats plans to toss a fair coin 10,000 times in the hope that it will lead him to a deeper understanding of the laws of probability. Which of the following statements is true?

(a) It is unlikely that Dr. Stats will get more than 5000 heads. no expected over long run prob 1/2
(b) Whenever Dr. Stats gets a string of 15 tails in a row, it becomes more likely that the next toss will be a head.
(c) The fraction of tosses resulting in heads should be close to 1/2.
(d) The chance that the 100th toss will be a head depends somewhat on the results of the first 99 tosses.
(e) All of the above statements are true.

T5.2. China has 1.2 billion people. Marketers want to know which international brands they have heard of. A large study showed that 62% of all Chinese adults have heard of Coca-Cola. You want to simulate choosing a Chinese at random and asking if he or she has heard of Coca-Cola. One correct way to assign random digits to simulate the answer is:

(a) One digit simulates one person's answer; odd means "Yes" and even means "No." This is 50/50
(b) One digit simulates one person's answer; 0 to 6 mean "Yes" and 7 to 9 mean "No." 70% is yes
(c) One digit simulates the result; 0 to 9 tells how many in the sample said "Yes." this is 100%
(d) Two digits simulate one person's answer; 00 to 61 mean "Yes" and 62 to 99 mean "No." yes 62% yes
(e) Two digits simulate one person's answer; 00 to 62 mean "Yes" and 63 to 99 mean "No."
63% yes

T5.3. Choose an American household at random and record the number of vehicles they own. Here is the probability model if we ignore the few households that own more than 5 cars:

Number of cars:	0	1	2	3	4	5
Probability:	0.09	0.36	0.35	0.13	0.05	0.02

A housing company builds houses with two-car garages. What percent of households have more cars than the garage can hold?

P(3 or 4 or 5) = .13 + .05 + .02 = .2

(a) 7% (b) 13% (c) 20% (d) 45% (e) 55%

T5.4. Computer voice recognition software is getting better. Some companies claim that their software correctly recognizes 98% of all words spoken by a trained user. To simulate recognizing a single word when the probability of being correct is 0.98, let two digits simulate one word; 00 to 97 mean "correct." The program recognizes words(or not) independently. To simulate the program's performance on 10 words, use these random digits:

60970 70024 17868 29843 61790 90656 87964 18883

The number of words recognized correctly out of the 10 is

(a) 10 (b) 9 (c) 8 (d) 7 (e) 6

60	97	07	00	24	17	86	82	98	43
1	2	3	4	5	6	7	8	9	10
C	C	C	C	C	C	C	C	~~C~~	C

9/10 correct

Questions T5.5 to T5.7 refer to the following setting. One thousand students at a city high school were classified according to both GPA and whether or not they consistently skipped classes. The two-way table below summarizes the data.

	GPA			
Skipped Classes	**<2.0**	**2.0–3.0**	**>3.0**	Total
Many	80	25	5	110
Few	175	450	265	890
	255	475	270	1000

T5.5. What is the probability that a student has a GPA under 2.0?

(a) 0.227 (b) 0.255 (c) 0.450 (d) 0.475 (e) 0.506

$P(GPA < 2.0) = \frac{255}{1000} = .255$

T5.6. What is the probability that a student has a GPA under 2.0 or has skipped many classes?

(a) 0.080 (b) 0.281 (c) 0.285 (d) 0.365 (e) 0.727

$P(GPA < 2.0 \text{ or many}) = \frac{255}{1000} + \frac{110}{1000} - \frac{80}{1000} = \frac{285}{1000} = .285$

T5.7. What is the probability that a student has a GPA under 2.0 given that he or she has skipped many classes?

(a) 0.080 (b) 0.281 (c) 0.285 (d) 0.314 (e) 0.727

$P(GPA < 2.0 \mid Many) = \frac{80}{110} = .727$

T5.8. For events *a* and *B* related to the same chance process, which of the following statements is true?

(a) If *a* and *B* are mutually exclusive, then they must be independent.
(b) If *a* and *B* are independent, then they must be mutually exclusive.
(c) If *a* and *B* are not mutually exclusive, then they must be independent.
(d) If *a* and *B* are not independent, then they must be mutually exclusive.
(e) If *a* and *B* are independent, then they cannot be mutually exclusive.

T5.9. Choose an American adult at random. The probability that you choose a woman is 0.52. The probability that the person you choose has never married is 0.25. The probability that you choose a woman who has never married is 0.11. The probability that the person you choose is either a woman or has never been married (or both) is therefore about

(a) 0.77 (b) 0.66 (c) 0.44 (d) 0.38 (e) 0.13

$P(W \text{ or never married}) = .52 + .25 - .11 = .66$

T5.10. A deck of playing cards has 52 cards, of which 12 are face cards. If you shuffle the deck well and turn over the top 3 cards, one after the other, what's the probability that all 3 are face cards?

(a) 0.001 (b) 0.005 (c) 0.010 (d) 0.012 (e) 0.02

$P(3 \text{ face cards}) = \underbrace{\frac{12}{52}}_{\text{1st face card}} \cdot \underbrace{\frac{11}{51}}_{\text{2nd face card}} \cdot \underbrace{\frac{10}{50}}_{\text{3rd face card}} = \frac{1320}{132,600} \approx .010$ (.00995)

Section II: Free Response *Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.*

5.11. Your teacher has invented a "fair" dice game to play. Here's how it works. Your teacher will roll one fair *eight-sided* die, and you will roll a fair *six-sided* die. Each player rolls once, and the winner is the person with the higher number. In case of a tie, neither player wins. The table shows the sample space of this chance process.

	Teacher Rolls							
You Roll	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	1,1	1,2 T	1,3 T	1,4 T	1,5 T	1,6 T	1,7 T	1,8 T
2	2,1	2,2	2,3 T	2,4 T	2,5 T	2,6 T	2,7 T	2,8 T
3	3,1	3,2	3,3	3,4 T	3,5 T	3,6 T	3,7 T	3,8 T
4	4,1	4,2	4,3	4,4	4,5 T	4,6 T	4,7 T	4,8 T
5	5,1	5,2	5,3	5,4	5,5	5,6 T	5,7 T	5,8 T
6	6,1	6,2	6,3	5,5	6,5	6,6	6,7 T	6,8 T

(a) Let *a* be the event "your teacher wins." Find *P*(*a*).

$P(a) = \frac{27}{48}$

(b) Let *B* be the event "you get a 3 on your first roll." Find *P*(*a* ∪ *B*)

$P(a \cup B) = P(a) + P(B) - P(a \cap B)$

$\frac{27}{48} + \frac{8}{48} - \frac{5}{48} = \frac{30}{48} = \frac{5}{8} = .625$

(c) Are events *a* and *B* independent? Justify your answer.

① $P(a) = \frac{27}{48}$ $\quad P(B|A) = \frac{5/48}{27/48} = 5/27 \quad P(B|A) \neq P(B)$

$\therefore$ not independent

$P(B) = \frac{8}{48}$

Or

✓ $P(A \cap B) = P(A) \cdot P(B)$

$\frac{5}{48} = \frac{27}{48}\left(\frac{8}{48}\right)$

no, $\therefore$ not independent

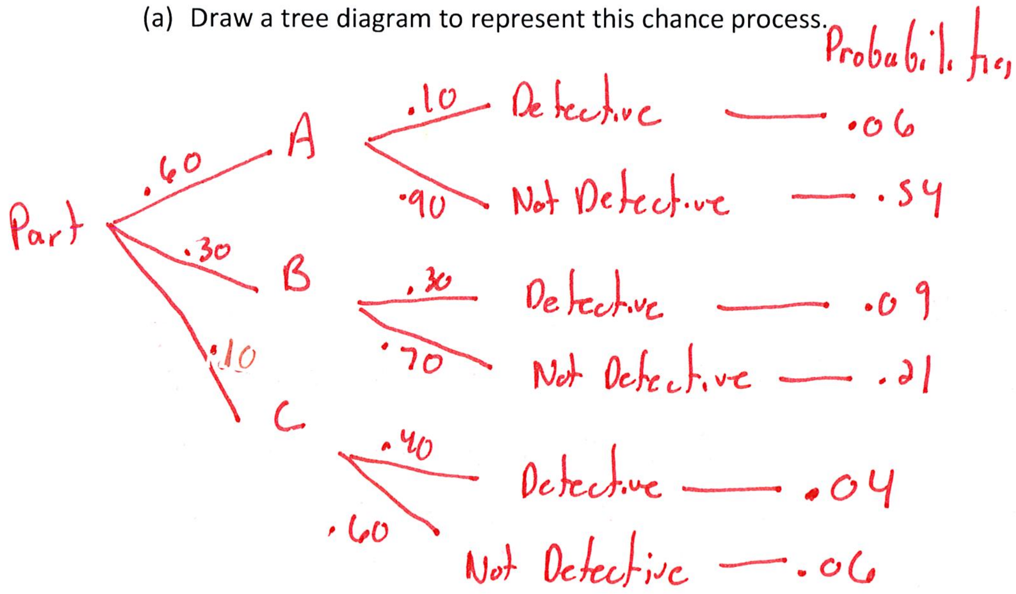
or

$P(A|B) = \frac{5/48}{8/48} = 5/8 \quad P(A|B) \neq P(A)$

$\therefore$ not independent

T5.12. Three machines—A, B, and C—are used to produce a large quantity of identical parts at a factory. Machine A produces 60% of the parts, while Machines B and C produce 30% and 10% of the parts, respectively. Historical records indicate that 10% of the parts produced by Machine A are defective, compared with 30% for Machine B and 40% for Machine C.

(a) Draw a tree diagram to represent this chance process.



(b) If we choose a part produced by one of these three machines, what's the probability that it's defective? Show your work.

$P(\text{Defective}) = .06 + .09 + .04 = .19$

(c) If a part is inspected and found to be defective, which machine is most likely to have produced it? Give appropriate evidence to support your answer.

$P(A \mid \text{Defective}) = \frac{.06}{.19} = .316$

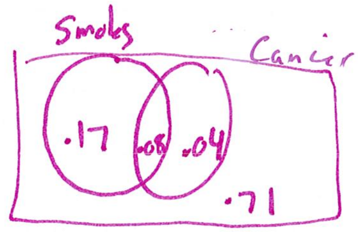
$P(B \mid \text{Defective}) = \frac{.09}{.19} = .474$

$P(C \mid \text{Defective}) = \frac{.04}{.19} = .211$

Machine B is most likely to have produced the part given it was defective.

T5.13. Researchers are interested in the relationship between cigarette smoking and lung cancer. Suppose an adult male is randomly selected from a particular population.The following table shows the probabilities of some events related to this chance process:

Event	Probability
Smokes	0.25
Smokes and gets cancer	0.08
Does not smoke and does not get cancer	0.71



(a) Find the probability that the individual gets cancer given that he is a smoker. Show your work.

P(Cancer | Smoker) $= \frac{.08}{.25} = .32$

(b) Find the probability that the individual smokes or gets cancer. Show your work.

P(Smokes or Cancer) = 1 − P(no smoke ∩ no cancer)

$= 1 - .71 = .29$

or from Venn Diagram P(S or C) $= .17 + .08 + .04 = .29$

(c) Two adult males are selected at random. Find the probability that at least one of the two gets cancer. Show your work.

P(at least one of the 2 get cancer) = 1 − P(neither get cancer)

P(getting cancer) $= .08 + .04 = .12$

for 1 person

so P (not getting cancer) $= 1 - .12 = .88$

for 1 person

P (not getting cancer for 2 people) $= .88^2 = .7744$

Prob (at least one of the 2 get cancer) $= 1 - .7744 = .2256$

T5.14. Based on previous records, 17% of the vehicles passing through a tollbooth have out-of-state plates. A bored tollbooth worker decides to pass the time by counting how many vehicles pass through until he sees two with out-of-state plates.[27]

(a) Describe the design of a simulation to estimate the average number of vehicles it takes to find two with out-of-state plates. Explain clearly how you will use the partial table of random digits below to carry out your simulation.

(b) Perform three repetitions of the simulation you described in part (a). Copy the random digits below onto your paper. Then mark on or directly above the table to show your results.

41050	92031	06449	05059	59884	31880
53115	84469	94868	57967	05811	84514
84177	06757	17613	15582	51506	81435
75011	13006	63395	55041	15866	06589

a) Assign #'s 01 through 17 to represent cars with out of state plates and the remaining #'s, 00 and 18-99, to represent the other cars. Start reading two digit numbers from random # table until you have found 2 #'s between 01 and 17. Record how many 2 digit #'s you read in order to get #'s between 01 and 17. Repeat many times for the simulation

b) Trial 1: 41 05 09   3 cars to find 2 out of state plates

Trial 2: 20 31 06 44 90 50 59 59 88 43 18 80 53 11 14 cars to find 2 out of state plates

Trial 3: 58 44 69 94 86 85 79 67 05 81 18 45 14

13 cars to find 3 out of state plates

A and B independent

to check for independence

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = P(B)$$

A and B Mutually Exclusive

to check if events are M.E

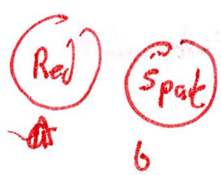
$$P(A \cap B) = 0$$

Problem 5. 8

A) If a and b M.E, then they must be independent.

Counterexample.

Let a: Red card in deck of cards b: Spade card in deck of cards



a and b are ME ✓

are they independent ?

$$P(R|S) = P(R)$$

$$P(R|S) = \frac{P(R \cap S)}{P(S)} = \frac{0}{13} = 0$$

$$P(R) = \frac{13}{52} = \frac{1}{4}$$

$P(R|S) \neq P(R)$ ∴ not independent

Counterexample

B) If a and B are independent, then they must be mutually exclusive.

Let a: Red Card B: Ace

$$P(R \cap A) = 2$$

∴ not mutually exclusive

$(P(R \cap A) \neq 0)$

$$P(R|A) = P(R)$$

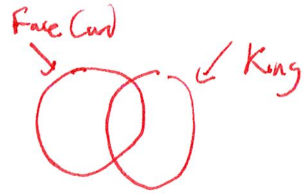
$$P(R|A) = \frac{P(R \cap A)}{P(A)} = \frac{2}{4} = \frac{1}{2}$$

$$P(R) = \frac{26}{52} = \frac{1}{2}$$ ∴ independent

c) If a and B are not Mutually Exclusive, then they must be independent

Counterexample

a: face card   b: king



not ME

$P(\text{Face Card} \cap \text{King}) = \frac{2}{52} \neq 0$

so not mutually exclusive

are a and b independent?

$$P(F|K) = \frac{P(F \cap K)}{P(K)} = \frac{\frac{4}{52}}{\frac{13}{52}} = \frac{4}{52} \cdot \frac{52}{13} = \boxed{\frac{4}{13}}$$

$$P(F) = \frac{13}{52} = \frac{1}{13}$$

$$\frac{1}{13} \neq \frac{4}{13}$$

∴ ~~not independent~~

D) use example above to show D is false

If a and b are not independent, then they must be mutually exclusive

E) If a and B are independent, then they cannot be mutually exclusive.

If a and B are independent, then $P(B|A) = \frac{P(A \cap B)}{P(A)} = P(B)$

and $P(A) \neq 0$, so $P(A) > 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

and $P(B) \neq 0$, so $P(B) > 0$

This means $P(A \cap B) = P(A) \cdot P(B) \neq 0$ since $P(A)$ and $P(B)$ both greater than

∴ A and B cannot be mutually exclusive