Lecture Notes & Examples 2.2 Part A

Section 2.2 –Normal Distributions

Probably the most famous of all *density curves* are **Normal curves**. The distributions they describe are called **Normal distributions**. They play a very large part in statistics.

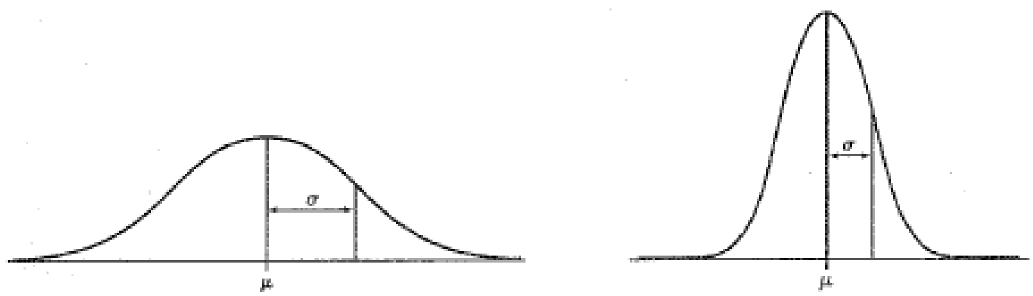


FIGURE 2.11 Two Normal curves, showing the mean μ and standard deviation σ.

Normal curves have several properties:

- All Normal curves have the same overall shape: symmetric, single-peaked, bell-shaped.
- Any specific Normal curve is completely described by its mean μ and standard deviation σ.
- The mean is located at the center and is equal to the median. Changing μ without changing σ moves the Normal curve along the horizontal axis without changing its shape.
- The standard deviation σ controls the spread of a Normal curve. Normal curves with larger standard deviations are more spread out.

The points at which the Normal curve changes from *concave down* to *concave up* occurs one standard deviation from the mean. Because of this, the standard deviation can be estimated by the graph.

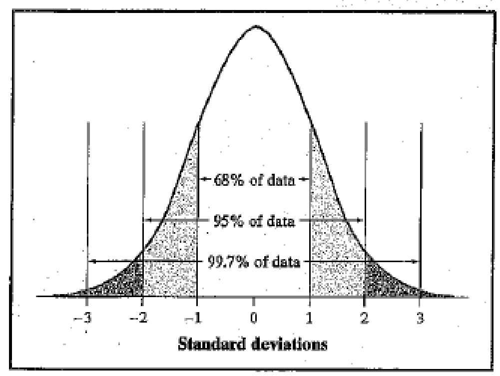
Definition: A **Normal distribution** is described by a Normal density curve. Any particular Normal distribution is completely specified by its mean μ and standard deviation σ. The mean of a Normal distribution is at the center of the symmetric **Normal curve** and equals the median. The standard deviation is the distance from the center to the inflection points (where concavity changes) on either side.

Notation: We abbreviate the Normal distribution with mean μ and standard deviation σ as ***N*(μ, σ)**.

The 68-95-99.7 Rule

In a Normal distribution with mean μ and standard deviation σ:

- Approximately 68% of the observations fall within 1 σ of the mean μ.
- Approximately 95% of the observations fall within 2 σ's of the mean μ.
- Approximately 99.7% of the observations fall within 3 σ's of the mean μ.



(Note: this rule does not apply to any distribution – only the Normal. Common error on AP Exam.)

Example: The mean batting average for the 432 Major League Baseball players in 2009 was 0.261 with a standard deviation of 0.034. Suppose the distribution is exactly Normal with μ = 0.261 and σ = 0.034.

a. Sketch a Normal density curve for this distribution. Label the points that are 1, 2, and 3 standard deviations from the mean.

b. What percent of batting averages are above 0.329?

c. What percent of batting averages are between 0.193 and 0.295?

CHECK YOUR UNDERSTANDING The distribution of heights of young women aged 18 to 24 is approximately N(64.5, 2.5).

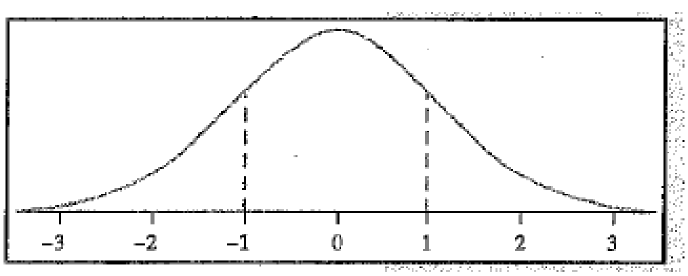
1. Sketch a Normal density curve for the distribution of young women's heights. Label the points one, two, and three standard deviations from the mean.

2. What percent of young women have heights greater than 67 inches? Show your work.

3. What percent of young women have heights between 62 and 72 inches? Show your work.

The Standard Normal Distribution

Definition: The **standard Normal distribution** is the Normal distribution with mean 0 and standard deviation 1. If a variable x has any Normal distribution *N*(μ, σ) with mean μ and standard deviation σ, then the standardized variable $z = \frac{x-\mu}{\sigma}$ has the standard Normal distribution.



68-95-99.7 Rule: For the standard Normal distribution

The **standard Normal table** is contained in Table A. It is a table of areas under the Normal curve. The table entry for each value z is the area under the curve to left of z. This is also known as the *lower tail.*

Table A *(Continued)* St

z	.00	.01	.02
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255

Example: Finding areas under the standard Normal curve.

Use *Table A* to find the proportion of observations from the standard Normal distribution given the following z-values. Draw a diagram for each.

a. Less than z = -1.25

b. Less than z = 0.81

c. Greater than z = 0.81

d. Between z = -1.25 and z = 0.81

Example: Repeat the previous example using *technology*.

a. Less than z = -1.25

b. Less than z = 0.81

c. Greater than z = 0.81

d. Between z = -1.25 and z = 0.81

Use -100 for lower bound when finding areas to the left in a **standard Normal distribution** since it's virtually impossible to be 100 standard deviations below the mean in a normal distribution.

Use 100 for upper bound when finding areas to the right in a **standard Normal distribution** since it's virtually impossible to be 100 standard deviations above the mean in a Normal distribution.

Example: Working backwards…..
Find the 90th percentile of standard Normal distribution

a) Using *Table A (Look in the body of table for entry closest to .90.)*

This is the entry corresponding to

b. Using *technology*

CHECK YOUR UNDERSTANDING

Use the z table (Table A in the back of the book) to find the proportion of observations from a standard Normal distribution that fall in each of the following regions. In each case, sketch a standard Normal curve and shade the area representing the region.

1. $z < 1.39$

2. $z > -2.15$

3. $-0.56 < z < 1.81$

4. The 20th percentile

5. 45% of all observations are greater than z