Lecture Notes & Examples 5.2

Section 5.2 - Probability Rules (pp. 299-308)

1. Definitions

- **Sample Space** - The set of all possible outcomes of a chance process

  EX) When you flip a coin, the sample space is {H, T}

- **Probability Model** – Description of some chance process that consists of two parts:
  A sample space S and probability for each outcome

  EX) When you flip a coin, S = {H, T} and P(Heads) = 1/2 and P(Tails) = 1/2

- **Event** – Any collection of outcomes from some chance process. An event is a subset of the sample space. Events are usually designated by capital letters.

  EX) When you flip a coin, Event A can be defined as Heads
  Event B can be defined as Tails

Mutually Exclusive – (Disjoint) Events that cannot occur at the same time – they have no outcome in common

le: 3 Flips of a Coin

S = { (H,H,H), (H,H,T), (H,T,T), (H,T,H), (T,T,T), (T,T,H) (T,H,H), (T,H,T) }

8 possible outcomes

A = 2 or more Heads B = No Heads

$P(A) = \frac{4}{8} = \frac{1}{2}$ $P(B) = \frac{1}{8}$

Events A and B are mutually exclusive

They have no outcomes in common (events cannot occur at the same time)

2. Basic Rules of Probability

- The probability of any event is A number between 0 and 1 inclusive

  $0 \le P(x_i) \le 1$

- All possible outcomes together must have probabilities that sum to 1

  $\sum P(x_i) = 1$

- If all possible outcomes in a sample space are equally likely, the probability that event A occurs can be found using the formula

  $$P(A) = \frac{\text{\# of outcomes in Event A}}{\text{Total \# of outcomes in Sample Space}}$$

- The probability that an event does not occur is

  1 − P (It Does Occur) ; P(Not A) = 1 − P(A) Not A ⇒ Compliment of A (A' or A^c)

- If two events have no outcomes in common, the probability that one or the other occurs is

  (Mutually Exclusive / Disjoint)

  The sum of their individual probabities

If A and B are Disjoint (M.E.), then

$$P(A \text{ or } B) = P(A) + P(B)$$

Basic Probability Rules

- For any event A, $0 \leq P(A) \leq 1$
- If S is the sample space in a probability model, $P(S) = 1$
- In the case of equally likely outcomes,

$$P(A) = \frac{number\ of\ outcomes\ corresponding\ to\ event\ A}{total\ number\ of\ outcomes\ in\ the\ sample\ space}$$

- Complement rule: $P(A^C) = 1 - P(A)$
- Addition rule for mutually exclusive events: If A and B are mutually exclusive,

$$P(A\ or\ B) = P(A) + P(B)$$

Example - Randomly select a student who took the 2013 AP Statistics exam and record the student's score. Here is the probability model:

Score	1	2	3	4	5
Probability	0.233	0.183	0.235	0.224	0.125

(a) Show that this is a legitimate probability model.

① All probabilities are between 0 and 1

② Sum of all probabilities are 1

∴ legit probability model

(b) Find the probability that the chosen student scored 3 or better.

P(3 or better) = 0.235 + 0.224 + 0.125 = 0.584

(c) Find the probability that the chosen student did not score a 1.

P (not score 1) = 1 - P(score 1)

= 1 - 0.233 = .767

CHECK YOUR UNDERSTANDING

Choose an American adult at random. Define two events:

A = the person has a cholesterol level of 240 milligrams per deciliter of blood (mg/dl) or above (high cholesterol)

B = the person has a cholesterol level of 200 to 239 mg/dl (borderline high cholesterol)

According to the American Heart Association, P(A) = 0.16 and P(B) = 0.29.

1. Explain why events A and B are mutually exclusive.

A person cannot be in both cholesterol zones at the same time

2. Say in plain language what the event "A or B" is. What is P(A or B)?

A person either has a cholesterol level of 240 or above or they have a cholesterol level between 200 and 239

$P(A \text{ or } B) = P(A) + P(B) = 0.16 + 0.29 = 0.45$

3. If C is the event that the person chosen has normal cholesterol (below 200 mg/dl), what's P(C)?

$P(C) = 1 - P(A \text{ or } B)$

$P(C) = 1 - 0.45$

$\boxed{P(C) = 0.55}$

3. Two-Way Tables and Probability

When we are trying to find probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easy.

Example - What is the relationship between educational achievement and home ownership? A random sample of 500 people who participated in the 2000 census was chosen. Each member of the sample was identified as a high school graduate (or not) and as a homeowner (or not). The two-way table displays the data.

	High School Grad	Not a HS Grad	Total
Homeowner	221	119	340
Not a homeowner	89	71	160
Total	310	190	500

Suppose we choose a member of the sample at random. Find the probability that the member

(a) is a high school graduate.

$$\frac{310}{500}$$

(b) is a high school graduate and owns a home.

$$\frac{221}{500}$$

c) is a high school graduate or owns a home.

P(h.s. grad) + P(owns a home) - P(hs grad & owns home)

$$\frac{310}{500} + \frac{340}{500} - \frac{221}{500} = \frac{429}{500}$$

or

$$\frac{221}{500} + \frac{119}{500} + \frac{89}{500} = \frac{429}{500}$$

General Addition Rule for Two Events

If A and B are any two events resulting from some chance process, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

"or" is $\cup$ "and" is $\cap$ *AP Exam Formula Sheet*

CHECK YOUR UNDERSTANDING

A standard deck of playing cards (with jokers removed) consists of 52 cards in four suits— clubs, diamonds, hearts, and spades. Each suit has 13 cards, with denominations ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king. The jack, queen, and king are referred to as "face cards." Imagine that we shuffle the deck thoroughly and deal one card. Let's define events A: getting a face card and B: getting a heart.

1. Make a two-way table that displays the sample space.

	Face Card	Not a Face Card	Total
Heart	3	10	13
Not a Heart	9	30	39
Total	12	40	52

2. Find P(A and B).

$P(A \cap B) = \frac{3}{52}$

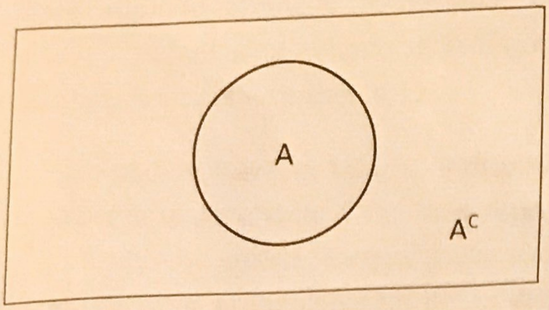
3. Explain why P(A or B) ≠ P(A) + P(B). Then use the general addition rule to find P(A or B).

Since A and B are not disjoint (not mutually exclusive) this is not equal to the sum of P(A) and P(B).

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

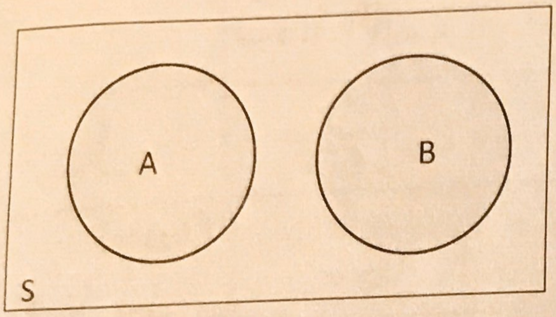
$\frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} \approx .423$

4. Venn Diagrams and Probability



Complementary Events

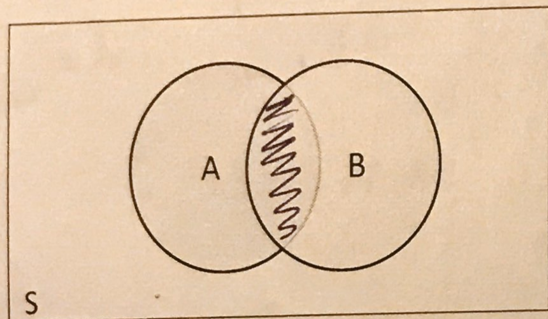
$P(A) + P(A^c) = 1$



Mutually Exclusive Events (no overlap)

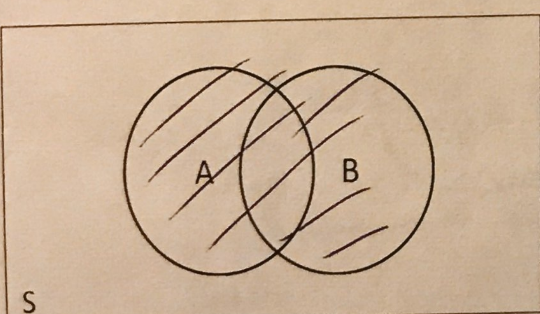
$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$



Not Disjoint Events (Not M.E.)

$P(A \cap B)$ = Area of overlap



$P(A \cup B)$ = Total Area

(Do Not Count Overlap Twice)

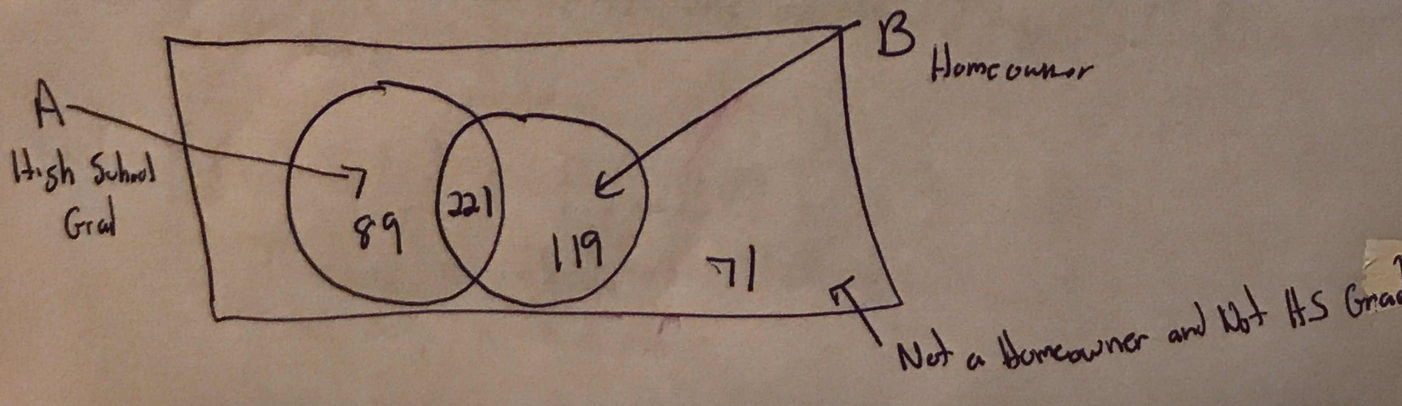
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B)$: Area of overlap

Examples – pp. 306-307 (Reference these pages for worked out examples.)

Now go back to the homeowner problem and create a Venn Diagram illustrating the two way table. (Given below)

	Event A High School Grad	Not a HS Grad	Total
Event B Homeowner	221	119	340
Not a homeowner	89	71	160
Total	310	190	500



Application: According to the National Center for Health Statistics, in December 2012, 60% of US households had a traditional landline telephone, 89% of households had cell phones, and 51% had both. Suppose we randomly selected a US household in December 2012.

a. Make a two-way table that displays the sample space of this chance process.
b. Construct a Venn diagram to represent the outcomes of this chance process.
c. Find the probability that the household has at least one of two types of phones.
d. Find the probability that the household has a cell phone only.

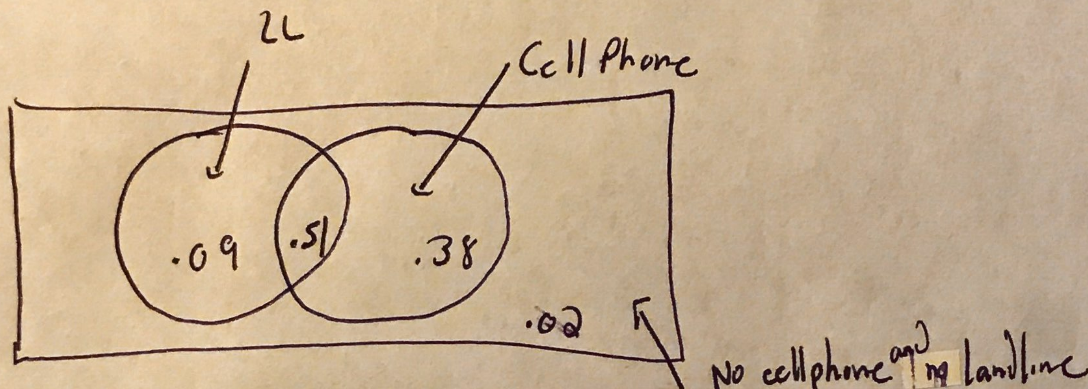
	Cell Phone	No Cell Phone (or Cell phone')	Total
LL	.51	.09	.60
NO LL (or LL')	.38	.02	.40
Total	.89	.11	1

$.60 - .51 = .09$

$.40 - .38 = .02$

$.89 - .51 = .38$

(B)



(C) $P(\text{at least one of 2 types}) = P(LL \cup CP) = P(LL) + P(CP) - P(LL \cap CP)$

$= .60 + .89 - .51$

$= .98$

∴ There is a .98 probability that the household has at least one of the 2 types of cell phones.

$P(LL \cup CP) = 1 - P(LL \cup CP)'$

$= 1 - .02$

$= .98$

D) $P(\text{cell only}) = P(\text{cell} \cap \text{no } LL) = .38$

There is a .38 prob. That the household has only a cell phone