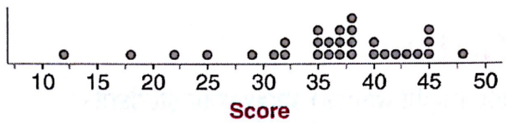
Lecture Notes & Examples 2.1 Part B

Section 2.1 (continued)

3. Transforming Data

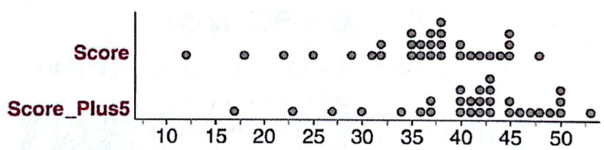
Example: Below is a graph and table of summary statistics for a sample of 30 test scores. The maximum possible score on the test was 50 points.



	n	$\bar{x}$	s_x	Min	Q_1	M	Q_3	Max	IQR	Range
Score	30	35.8	8.17	12	32	37	41	48	9	36

Suppose that the teacher was *nice* and added 5 points to each test score. How would this change the shape, center, and spread of the distribution?

Here are the graphs and the summary statistics for the original scores and the +5 scores:



	n	$\bar{x}$	s_x	Min	Q_1	M	Q_3	Max	IQR	Range
Score	30	35.8	8.17	12	32	37	41	48	9	36
Score + 5	30	40.8	8.17	17	37	42	46	53	9	36

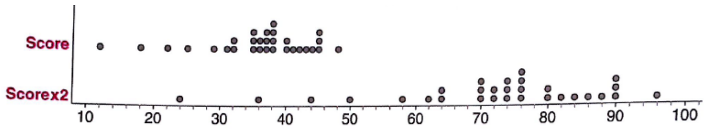
Effect of Adding (or Subtracting) a Constant
Adding the same number *a* (either positive, zero, or negative) to each observation:

- Adds *a* to measures of center and location (mean, median, quartiles, percentiles), but
- Does not change the *shape* of the distribution or measures of spread (range, IQR, standard deviation.

Application: If 24 is added to every observation in a data set, the only one of the following that is *not* changed is:

(a) the mean (b) the 75th percentile (c) the median (d) the standard deviation (e) the minimum

Example (cont): Suppose that the teacher in the previous example wanted to convert the *original* test scores to percents. Since the test was out of 50 points, he should multiply each score by 2 to make them out of 100. Here are the graphs and summary statistics for the original scores and the doubled scores.



	n	$\bar{x}$	s_x	Min	Q_1	M	Q_3	Max	IQR	Range
Score	30	35.8	8.17	12	32	37	41	48	9	36
Score × 2	60	71.6	16.34	24	64	74	82	96	18	72

What happened the measures of center, location and spread?

What happened to the shape?

Effect of Multiplying (or Dividing) by a Constant
Multiplying (or dividing) each observation by the same number b (positive, negative or 0)

- Multiplies (divides) measures of *center, location* (mean, median, quartiles, percentiles) by b,
- Multiplies (divides) measures of *spread* (range, IQR, standard deviation) by $|b|$, but
- Does not change the *shape* of the distribution.

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4. Transformations and Z-Scores

Example (continued). Suppose we wanted to standardize the original test scores. This would mean we would subtract the mean of 35.8 from each score and then divide by the standard deviation of 8.17.

	n	$\bar{x}$	s_x	Min	Q_1	M	Q_3	Max	IQR	Range
Score	30	35.8	8.17	12	32	37	41	48	9	36

What effect would these transformations have on:

- Shape?
- Center?
- Spread?

CHECK YOUR UNDERSTANDING

The figure below shows a dotplot of the height distribution for Mrs. Navard's class, along with summary statistics from computer output.

Variable	n	$\bar{x}$	s_x	Min	Q_1	M	Q_3	Max
Height	25	67	4.29	60	63	66	69	75

1. Suppose that you convert the class's heights from inches to centimeters (1 inch = 2.54 cm). Describe the effect this will have on the shape, center, and spread of the distribution.

2. If Mrs. Navard had the entire class stand on a 6-inch-high platform and then had the students measure the distance from the top of their heads to the ground, how would the shape, center, and spread of this distribution compare with the original height distribution?

3. Now suppose that you convert the class's heights to z-scores. What would be the shape, center, and spread of this distribution? Explain.

5. Density Curves

Exploring Quantitative Data
1. Always plot your data: make a graph, usually a dotplot, stemplot or a histogram.
2. Look for the overall pattern (shape, center, spread) and for striking departures such as outliers.
3. Calculate a numerical summary to briefly describe center and spread.

New step:
4. Sometimes the overall pattern of a *large* number of observations is so regular that we can describe it with a *smooth curve*.

This type of *smooth curve* is called a **Density Curve**.

Definition: A **density curve** is a curve that

- Is always above the horizontal axis, and
- Has an area of exactly 1 underneath it

A density curve describes the overall pattern of a distribution. The area under the curve and above any interval of values on the horizontal axis is the proportion of all observations that fall in that interval.

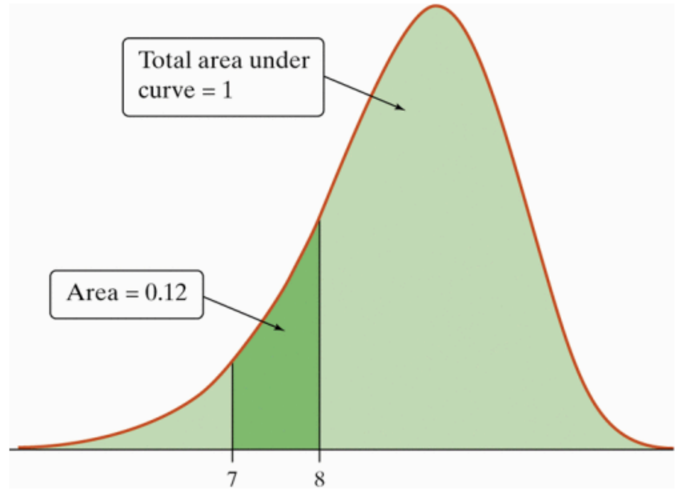
Note: *no set of real data is exactly described by a density curve. The curve is an approximation that is easy to use and accurate enough for practical use.*

Because the density curve represents a *population* of individuals, the mean is denoted by μ (the Greek letter mu) and the standard deviation is denoted by σ (the Greek letter sigma).

Distinguishing the Median and Mean of a Density Curve (Diagrams on p. 102)

- The **median** of a density curve is the *equal-areas* point, the point that divides the area under the curve in half.
- The **mean** of a density curve is the *balance point*, the point at which the curve would balance if made of solid material.
- The median and mean are the same for a perfectly symmetric density curve. They both lie at the center of the curve. The mean of a skewed curve is pulled away from the median in the direction of the long tail.

CHECK YOUR UNDERSTANDING
Use the figure shown to answer the following questions.



1. Explain why this is a legitimate density curve.

2. About what proportion of observations lie between 7 and 8?

3. Trace the density curve onto your paper. Mark the approximate location of the median.

4. Now mark the approximate location of the mean. Explain why the mean and median have the relationship that they do in this case.