2. The manager of a local fast-food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.

(a) Construct and interpret a 95 percent confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.

(b) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant $0.25. Suppose that in the month of June 3,000 customers ask for a water cup when placing an order. Use the confidence interval constructed in part (a) to give an interval estimate for the cost to the restaurant for the month of June from the customers who ask for a water cup but fill the cup with a soft drink.

a)

State: We want to estimate the true proportion p of all customers who ask for a water cup, but fill it up with a soft drink at a 95% confidence level.

Plan: Use one sample z-interval for p

① Random: Random Sample of 80 customers ✓

② Independent (10%): $10(80) < $ pop of all customers who ask for water cup
800
Reasonable to assume ✓

③ Normal (Large Counts) $23 \geq 10$ and $57 \geq 10$ ✓
# of successes fill cup with soft drink
# of failures fill cup with water

DO: PE ± ME → $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ → $.2875 \pm 1.96\sqrt{\frac{.2875(.7125)}{80}}$

$\hat{p} = \frac{23}{80} = .2875$

$z^*_{95\%} = 1.96$

→ $.2875 \pm .0992$ → $(.1883, .3867)$

Conclude: We are 95% confident the the interval from .1883 to .3867 captures the true proportion of all customers at this fast food restaurant who ask for a water cup and fill it with a soft drink.

b) .1883 of 3000 customers is 564.9, and .3867 of 3000 is 1160.1
Using the CI in Part A, a 95% interval estimate for # of customers in June who ask for water cup but filled it with a soft drink varies from 565 to 1160. At a cost of \$.25 per customer, a 95% interval estimate of cost to the restaurant in June is (\$141.25, \$290.00)

2. To increase business, the owner of a restaurant is running a promotion in which a customer's bill can be randomly selected to receive a discount. When a customer's bill is printed, a program in the cash register randomly determines whether the customer will receive a discount on the bill. The program was written to generate a discount with a probability of 0.2, that is, giving 20 percent of the bills a discount in the long run. However, the owner is concerned that the program has a mistake that results in the program not generating the intended long-run proportion of 0.2.

The owner selected a random sample of bills and found that only 15 percent of them received discounts. A confidence interval for p, the proportion of bills that will receive a discount in the long run, is 0.15 ± 0.06. All conditions for inference were met.

$\hat{p} \pm ME$

(a) Consider the confidence interval 0.15 ± 0.06.

(i) Does the confidence interval provide convincing statistical evidence that the program is not working as intended? Justify your answer.

(ii) Does the confidence interval provide convincing statistical evidence that the program generates the discount with a probability of 0.2 ? Justify your answer.

Orig sample n; ~~orig ME~~ $= z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

A second random sample of bills was taken that was four times the size of the original sample. In the second sample 15 percent of the bills received the discount. 4n

(b) Determine the value of the margin of error based on the second sample of bills that would be used to compute an interval for p with the same confidence level as that of the original interval.

$= z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{4n}}$

$\frac{z^* \sqrt{\hat{p}(1-\hat{p})}}{2\sqrt{n}}$

(c) Based on the margin of error in part (b) that was obtained from the second sample, what do you conclude about whether the program is working as intended? Justify your answer.

so orig ME / 2

a) (i) The confidence interval does not provide convincing statistical evidence that the program is not working as intendes because .20 is within the interval (.09, .21), so the program could be discounting 20% off bills.

(ii) No. The confidence interval includes values from .09 to .21, so any value in that interval is a plausible value for the probability that the computer is using to generate discounts

b) When n is 4x larger the M.E. is ÷ by 2. This is because the SE is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$, so when we use 4n, then $SE = \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{4n}} = \frac{\sqrt{\hat{p}(1-\hat{p})}}{2\sqrt{n}}$, therefore the original ME of .06 is ÷2 to get .03.

c) The CI is now .15 ± .03 → (.12, .18). Since the entire interval is now below .20, we now have evidence to suggest the program is not working

GO ON TO THE NEXT PAGE.

STATISTICS
SECTION II
Part A
Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. An environmental group conducted a study to determine whether crows in a certain region were ingesting food containing unhealthy levels of lead. A biologist classified lead levels greater than 6.0 parts per million (ppm) as unhealthy. The lead levels of a random sample of 23 crows in the region were measured and recorded. The data are shown in the stemplot below.

Lead Levels

2	8
3	0
3	5 8 8
4	1 1 2
4	6 8 8
5	0 1 2 2 3 4
5	9 9
6	3 4
6	6 8

Key: 2|8 = 2.8 ppm

(a) What proportion of crows in the sample had lead levels that are classified by the biologist as unhealthy?

$\hat{p} = 4/23 \approx .1739$

$\bar{x}$ S_x

(b) The mean lead level of the 23 crows in the sample was 4.90 ppm and the standard deviation was 1.12 ppm. Construct and interpret a 95 percent confidence interval for the mean lead level of crows in the region.

State: We want to estimate the true mean μ lead level of all crows in this region at a 95% confidence level.

Plan: One sample t-interval for μ ① Random: Random sample of 23 crows

② Independent (10%): 10(23) < pop of all crows in this region; Reasonable to assume

230

③ Normal: n=23 23 ≱ 30, but no clear outliers or strong skewness in stem + leaf plot, ∴ Normal

Do: PE ± ME → $\bar{x} \pm t^*\left(\frac{S_x}{\sqrt{n}}\right)$ → $4.90 \pm 2.074\left(\frac{1.12}{\sqrt{23}}\right)$ → $4.90 \pm .484$

→ (4.416, 5.384)

$t^*_{95\%}$ → DF = 22 → invT(.025, 22) = 2.074

Conclude: We are 95% confident that the interval from 4.416 to 5.384 captures the true mean lead level of all crows in this region.

GO ON TO THE NEXT PAGE.