Lecture Notes & Examples 6.1

Section 6.1 Discrete and Continuous Random Variables (pages 340 – 352)

1. Random Variables. Consider tossing a fair coin 3 times. The sample space would be:

S = { HHH, HHT, HTH, THH, HTT, THT, TTH, TTT }

Let **X** represent the number of heads obtained. We can depict this situation in a **probability distribution of X**:

Value	0	1	2	3
Probability				

We can use the probability distribution to answer questions about the variable X such as what is $P(X \geq 1)$?

Definition: A **random variable** takes numerical values that describe the outcomes of some chance process. The **probability distribution** of a random variable gives its possible values and their probabilities.

2. Discrete Random Variables

Definition: A **discrete random variable** X takes on a fixed set of possible values with gaps between. The probability distribution of a discrete random variable X lists the values x_i and their probabilities p_i:

Value:	x_1	x_2	x_3	. . .
Probability:	p_1	p_2	p_3	. . .

The probabilities p_i must satisfy two requirements:

1. Every probability p_i is a number between 0 and 1.
2. The sum of the probabilities is 1.

To find the probability of any event, add the probabilities p_i of the particular values of x_i that make up that event.

Example - In 2010, there were 1319 games played in the National Hockey League's regular season. Imagine selecting one of these games at random and then randomly selecting one of the two teams that played in the game. Define the random variable X = number of goals scored by a randomly selected team in a randomly selected game. The table below gives the probability distribution of X:

Goals	0	1	2	3	4	5	6	7	8	9
Probability	0.061	0.154	0.228	0.229	0.173	0.094	0.041	0.015	0.004	0.001

(a) Show that the probability distribution for X is legitimate.

(b) What is the probability that the number of goals scored by a randomly selected team in a randomly selected game is at least 6?

Check Your Understanding

North Carolina State University posts the grade distributions for its courses online.[3] Students in Statistics 101 in a recent semester received 26% A's, 42% B's, 20% C's, 10% D's, and 2% F's. Choose a Statistics 101 student at random. The student's grade on a four-point scale (with A = 4) is a discrete random variable X with this probability distribution:

Value of X:	0	1	2	3	4
Probability:	0.02	0.10	0.20	0.42	0.26

1. Say in words what the meaning of $P(X \geq 3)$ is. What is this probability?

2. Write the event "the student got a grade worse than C" in terms of values of the random variable X. What is the probability of this event?

3. Sketch a graph of the probability distribution. Describe what you see.

3. The Mean (Expected Value) of a Discrete Random Variable

When analyzing shapes of distributions we used **SOCS**. If we want to know the center of a distribution of a discrete random variable we are going to have to compute the mean. The mean of a discrete random variable X is denoted by μ_x. It is an average of all possible values of the random variable X but we have to take into account how many times we *expect* the values to occur. For this reason the mean is also referred to as the **expected value** of the random variable.

Example: Given the probability distribution of the discrete random variable X, find the expected value of X.

Value	1	2	3
Probability	0.5	0.2	0.3

Definition: Suppose that X is a discrete random variable whose probability distribution is

Value:	x_1	x_2	x_3	. . .
Probability:	p_1	p_2	p_3	. . .

To find the **mean (expected value)** of X, multiply each possible value by its probability then add all the products:

Example: Find the expected value of the random variable X in the NHL example and interpret the value in context.

Goals	0	1	2	3	4	5	6	7	8	9
Probability	0.061	0.154	0.228	0.229	0.173	0.094	0.041	0.015	0.004	0.001

Note: *A common error on the AP Exam is that students incorrectly believe that the expected value of a random variable must be equal to one of the possible values of the variable. This is not the case.*

4. The Standard Deviation (and Variance) of a Discrete Random Variable

In order to describe the *spread* of the distribution of a discrete random variable, we are going to use the standard deviation. In order to find the standard deviation, we first compute the variance and then find its square root. The variance is the *average of the squared deviation of the possible X values from the mean*. Again, however, we must take into account how often we expect the different values of X to occur.

Definition: Suppose that X is a discrete random variable whose probability distribution is

Value:	x_1	x_2	x_3	. . .
Probability:	p_1	p_2	p_3	. . .

and that μ_x is the mean of X. The **variance of X** is

The **standard deviation** of X, σ_x is the square root of the variance.

Example. Compute and interpret the standard deviation of the random variable X in the NHL example and interpret its meaning in context.

Goals	0	1	2	3	4	5	6	7	8	9
Probability	0.061	0.154	0.228	0.229	0.173	0.094	0.041	0.015	0.004	0.001

Check Your Understanding

A large auto dealership keeps track of sales made during each hour of the day. Let X = the number of cars sold during the first hour of business on a randomly selected Friday. Based on previous records, the probability distribution of X is as follows:

Cars sold:	0	1	2	3
Probability:	0.3	0.4	0.2	0.1

1. Compute and interpret the mean of X.

2. Compute and interpret the standard deviation of X.

5. Continuous Random Variables

Definition: A **continuous random variable** X takes all values in an interval of numbers. The probability distribution of X is described by a *density curve*. The probability of any event is the area under the density curve and above the values of X that make up that event.

The most familiar continuous probability distribution is the (vaunted) **Normal distribution**.

Example: The weights of three-year-old females closely follow a *Normal distribution* with a mean of $\mu = 30.7$ pounds and a standard deviation of 3.6 pounds. Randomly choose one three-year-old female and call her weight X. Find the probability that the randomly selected female weighs at least 30 pounds.

State:

Plan:

Do:

Conclude: