10 random students' hours of sleep: 6, 7, 8, 5, 8, 9, 8, 4, 6.5, 9

Name: ______________________ Hour: _____ Date: ______

I ♥ SLEEP

Learning Targets

- State and check the Random, 10%, and Normal/Large Sample conditions for performing a significance test about a population mean.
- Calculate the standardized test statistic and P-value for a test about a population mean.
- Perform a significance test about a population mean.

Lesson 9.3: Day 1: Are you getting enough sleep?

It's recommended that teenagers get 8 hours of sleep a night. Mrs. Cowells believes her AP Stats students are getting less than the recommended 8 hours of sleep per night. To test her belief, take a random sample of 10 students in class and record the number of hours of sleep for each. Do these data provide convincing evidence that the AP stats students get less than 8 hours of sleep per night using α = 0.05?

1. Calculate the sample mean and standard deviation. (use calc)

$\bar{x} = 6.55$ $\quad S_x = 1.771$

2. State the appropriate hypotheses for a significance test. Be sure to define the parameter of interest.

$H_0: \mu = 8$ $\quad \mu$ = true mean hours of sleep for AP Stat Students at HFII

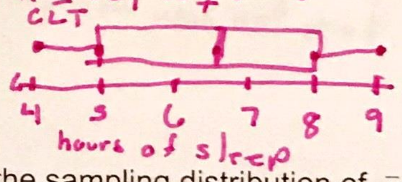
$H_a: \mu < 8$

3. What conditions must be met? Check them.

① Random
Random Sample of 10 ✓

② 10% (Independent)
$10(10) <$ pop of all AP Stat Students at HFII
100
no! (proceed with caution)

③ Normal
$n \geq 30$, $10 \ngeq 30$ no
CLT
no strong skew or outliers ✓



4. Give the formulas for the mean and standard deviation of the sampling distribution of $\bar{x}$ and calculate the values.

$\mu_{\bar{x}} = \mu$

$t = \dfrac{\bar{x} - \mu}{s_x/\sqrt{n}}$

$\mu_{\bar{x}} = \mu = 8$ $\quad \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} \approx \dfrac{s_x}{\sqrt{n}}$ $\quad \sigma_{\bar{x}} \approx \dfrac{s_x}{\sqrt{n}}$

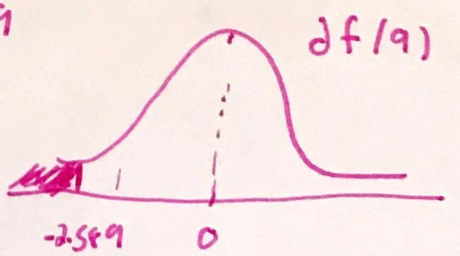
$\sigma_{\bar{x}} = \dfrac{1.771}{\sqrt{10}} \approx 0.56$

5. Draw a picture and then calculate the test statistic. t

degrees of freedom = $n-1$

$t = \dfrac{\bar{x} - \mu}{s_x/\sqrt{n}} = \dfrac{6.55 - 8}{.56} \approx -2.589$

df (9)



TheStatsMedic

Name: ______________________________ Hour: _____ Date: ______

6. Remember, since we are working with means, the test statistic is a t value. Use table B to find the P-value.

P-value from Table B between .01 and .02

Using Table B: Go to row $df = n-1$ → find closest t-values and match tail probability: P-value will be a range between 2 #'s.

* Use Calc

tcdf (lower, upper, df)

tcdf (-100, -2.589, 9) ≈ .015

7. What conclusion can we make?

P-Value of .015 < $\alpha = .05$, so we reject null hypothesis and have convincing evidence that the true mean hours of sleep for HF II AP Stat students is less than 8 hours. We must be cautious of our results, as the Independence condition was not satisfied.

Lesson 9.3 Day 1– Significance Test for μ

Important ideas:

L.T. #1 Conditions:

① Random ② Independence (10%) $10n < N$ ③ Normal
- Population is Normal
- $n \geq 30$ CLT
- No strong skew or outliers

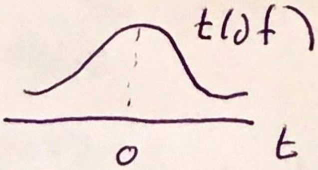
* Must Graph to show evidence *

*If a condition is not met, continue test and after conclusion state "we must be cautious of our results, as ___ condition was not met.

L.T. #2 Test Statistic

One Sample t test

$$t = \frac{\bar{x} - \mu}{s_x/\sqrt{n}}$$



L.T. #3 P-value

- Use table B with df and tail probability to find range for P-value

or

- Use calculator tcdf (lower, upper, df)
- for 2 sided significance test, p-value = 2 · tcdf (lower, upper, df)

Name: ______________________________ Hour: _____ Date: ______

Check Your Understanding

The level of dissolved oxygen (DO) in a stream or river is an important indicator of the water's ability to support aquatic life. A researcher measures the DO level at 30 randomly chosen locations along a stream. Here are the results in milligrams per liter (mg/l): $\bar{x} = 4.77$ and $s_x = 0.939$. An average dissolved oxygen level below 5 mg/l puts aquatic life at risk. Do the data provide convincing evidence at the α = 0.05 significance level that aquatic life in this stream is at risk?

State: Parameter: μ = true mean DO level

Statistic: $\bar{x} = 4.77$

Hypotheses: $H_0: \mu = 5$

$H_a: \mu < 5$

α Level: $\alpha = .05$

Plan: Name of procedure: One Sample t-test for μ

Check conditions:

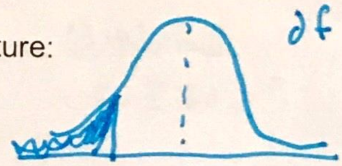
① Random
30 Randomly Chosen ✓

② Independence (10%)
10(30) ≤ pop all locations along stream ✓
300
Reasonable to assume

③ Normal
30 ≥ 30 by CLT
Normal ✓

Do: General: $t = \frac{\text{stat} - \text{null}}{\text{SD}}$

Picture:



Specific: $t = \frac{\bar{x} - \mu}{s_x/\sqrt{n}}$

Work: $t = \frac{4.77 - 5}{.939/\sqrt{30}} \approx -1.34$

Test Statistic: $t \approx -1.34$

P-value: Between .05 and .10

t cdf (-100, -1.34, 29) ≈ .095

Interpret P-Value: Assuming the mean DO level is 5 mg/l, there is a .095 probability of getting a sample mean of 4.77 mg/l or less purely by chance.

Conclude:

Since P-value of .095 > α = .05, we fail to reject the null and do not have convincing evidence that the true μ DO level is less than 5 mg/l.

Based on your conclusion, what type of error (Type I or Type II) could you have made? Describe the error in context and name a consequence of the error.

Type II b/c we failed to reject H_0

Type II: Ha is true, fail to reject Ho.

Determine the average DO is less than 5 mg/l, when it is not.

Consequence is spending a lot of money and unnecessary worry to try and fix D.O levels

Name: ______________________________________ Hour: _____ Date: ______

Two sided significance tests

In the children's game Don't Break the Ice, small plastic ice cubes are squeezed into a square frame. Each child takes turns tapping out a cube of "ice" with a plastic hammer hoping that the remaining cubes don't collapse. For the game to work correctly, the cubes must be big enough so that they hold each other in place in the plastic frame but not so big that they are too difficult to tap out. The machine that produces the plastic ice cubes is designed to make cubes that are 29.5 millimeters (mm) wide, but the actual width varies a little. To make sure the machine is working well, a supervisor inspects a random sample of 50 cubes every hour and measures their width. The output summarizes the data from a sample taken during one hour.

$s_x/\sqrt{n} = SE_{mean}$

variable	N	Mean	SEmean	stdDev	Min	Q1	Median	Q3	Max
Width	50	29.4874	0.0132	0.0935	29.2717	29.4225	29.4821	29.5544	29.7148

Do these data give convincing evidence that the mean width of cubes produced this hour is **not** 29.5 mm?

State: Parameter: μ = true mean width of the "ice cubes"

$H_0: \mu = 29.5$

$H_a: \mu \neq 29.5$

Significance level: $\alpha = .05$

statistic: $\bar{x} = 29.4874$

Plan: 1 sample t-test for μ

① Random
Random Sample of 50 ✓

② Independent (10%)
$10(50) \leq$ Pop of all ice cubes produced ✓
500
Reasonable to assume

③ Normal
$50 \geq 30$ ✓ by CLT

Do: test stat = $\frac{\text{stat} - \text{null}}{\text{SD}}$

df (49)

$$t = \frac{\bar{x} - \mu}{s_x/\sqrt{n}} = \frac{29.4874 - 29.5}{.0132} = \frac{-0.0126}{.0132} = .9545$$

−.9545 .9545

p value: $2 \cdot \text{tcdf}(-100, -.9545, 49) = .345$

Conclude: Since Pvalue of $.345 > \alpha = .05$, we fail to reject the null hypothesis. There is not convincing evidence that the true width of the ice cubes produced this hour is different from 29.5S.

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

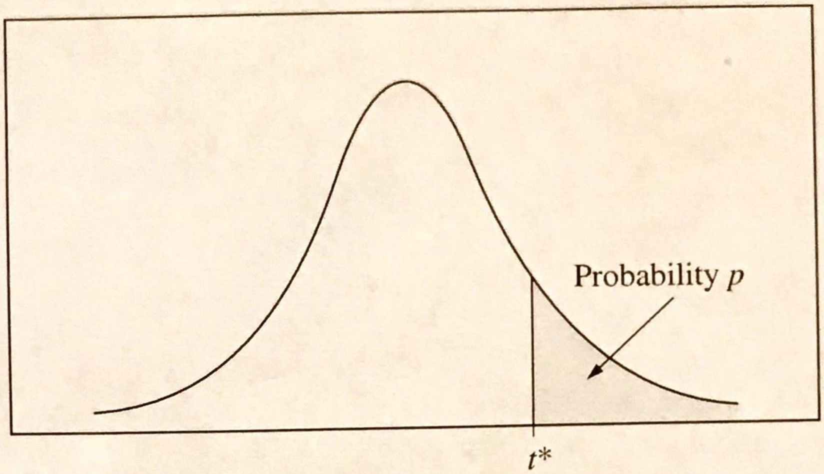


Table B ***t* distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

III. Sampling Distributions and Inferential Statistics (*continued*)

Sampling distributions for proportions:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For one population: $\hat{p}$	$\mu_{\hat{p}} = p$	$\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$	$s_{\hat{p}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$
For two populations: $\hat{p}_1 - \hat{p}_2$	$\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2$	$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$	$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ When $p_1 = p_2$ is assumed: $s_{\hat{p}_1-\hat{p}_2} = \sqrt{\hat{p}_c(1-\hat{p}_c)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}$ where $\hat{p}_c = \dfrac{X_1 + X_2}{n_1 + n_2}$

Sampling distributions for means:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For one population: $\overline{X}$	$\mu_{\overline{X}} = \mu$	$\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}}$	$s_{\overline{X}} = \dfrac{s}{\sqrt{n}}$
For two populations: $\overline{X}_1 - \overline{X}_2$	$\mu_{\overline{X}_1-\overline{X}_2} = \mu_1 - \mu_2$	$\sigma_{\overline{X}_1-\overline{X}_2} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$	$s_{\overline{X}_1-\overline{X}_2} = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

Sampling distributions for simple linear regression:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For slope: b	$\mu_b = \beta$	$\sigma_b = \dfrac{\sigma}{\sigma_x\sqrt{n}}$, where $\sigma_x = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n}}$	$s_b = \dfrac{s}{s_x\sqrt{n-1}}$, where $s = \sqrt{\dfrac{\Sigma(y_i - \hat{y}_i)^2}{n-2}}$ and $s_x = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n-1}}$

*Standard deviation is a measurement of variability from the theoretical population. Standard error is the estimate of the standard deviation. If the standard deviation of the statistic is assumed to be known, then the standard deviation should be used instead of the standard error.