

Learning Targets

- Interpret a Type I error and a Type II error in context. Give a consequence of each error in a given setting.

Lesson 9.1: Day 2: Should Rockford switch to bottled water?



WOLVERINE



The Wolverine Worldwide (a shoe company in Rockford) improperly disposed of chemicals (PFAS), which have leaked into the ground water. The state of Michigan says that if more than 7% of households in a city exceed the safe limit, the city needs to switch to bottled water. A concerned citizen takes a random sample of 100 households and finds that 12 have unsafe water. Do the data provide convincing evidence that Rockford should switch to bottled water?

1. State appropriate hypotheses for performing a significance test. Use $\alpha = 0.05$.

$p = 12/100 = 0.12$

$H_0: p = .07$ → The water is at the safe limit.
 $H_a: p > .07$ → the water is unsafe.
 p → true proportion of households in Rockford with unsafe water.

2. (a) After conducting a significance test, a P-value of 0.025 is found. Interpret this value.

Assuming H_0 is true ($p = .07$), there is a .025 probability of getting a \hat{p} of 0.12 or greater purely by chance.

- (b) Based on the P-value, should Rockford keep the current water or switch to bottled water?

Explain. Since P value of 0.025 < α of 0.05, we reject H_0 and have convincing that the true proportion of households is greater than 7%.

- (c) Let's suppose this decision is wrong. What would be a consequence of this error?

Consequence of this error would be spending a lot of money purchasing bottled water when we didn't need it. Type I Error → H_0 is true, Reject H_0

- (d) If the water is safe, what is the probability that this error will occur?

5% of the time we get statistically significant results (meaning unlikely to happen by chance) purely by chance.

3. (a) Now suppose the P-value was 0.217. In this case, would the city keep the current water or switch to bottled water?

Since P value of 0.217 > 0.05 we fail to reject H_0 , not convincing evidence against H_0 .

- (b) Let's suppose this decision is wrong. What would be a consequence of this error?

Many people would drink unsafe water and could get sick, even die.

4. Are the consequences in question #2 or question #3 more serious? Explain.

3, people would get very sick which is ^{much} worse than wasting money.

Lesson 9.1 Day 2 – Type 1 and Type 2 Errors

Important ideas:

		Truth	
		H_0 True	H_a True
Conclusion	Reject H_0	Type I Error	☺
	Fail to Reject H_0	☺	Type II

Type I Error: H_0 true. Reject H_0

"The null hypothesis is true, but
wrong decision

$$P(\text{type I}) = \alpha$$

Type II Error: - H_a true. Fail to reject H_0 .

"The alternative hypothesis is true, But
..... wrong decision.

Check Your Understanding

The manager of a fast-food restaurant wants to reduce the proportion of drive-thru customers who have to wait longer than 2 minutes to receive their food after placing an order. Based on store records, the proportion of customers who had to wait longer than 2 minutes was $p = 0.63$. To reduce this proportion, the manager assigns an additional employee to drive-thru orders. During the next month, the manager collects a random sample of 250 drive-thru times and finds that $\hat{p} = \frac{144}{250} = 0.576$. The manager then performs a test of the following hypotheses at the $\alpha = 0.10$ significance level:

$$H_0: p = 0.63 \quad \text{Customer Service did not improve}$$

$$H_a: p < 0.63 \quad \text{Customer Service Improve}$$

where p = the true proportion of drive-thru customers who have to wait longer than 2 minutes to receive their food.

1. Describe a Type I error and a Type II error in this setting.

Type I Error: H_0 True, Reject H_0

63% of customers wait longer than 2 minutes, but the manager finds convincing evidence that the additional employee reduced this proportion.

Type II Error: H_a True, Fail to Reject H_0

Less than 63% of customers wait longer than 2 min, but manager does not find convincing evidence that additional employee reduced the proportion.

2. Which type of error is more serious in this case? Justify your answer.

Type I - manager is spending money on extra employee believing it reduces the proportion of customers who wait, but it does not.

(Some students might say Type 2 - b/c he might fire employee since he did not find evidence of the employee reducing proportion of customers who wait longer than 2 min)

3. Based on your answer to Question 2, do you agree with the company's choice of $\alpha = 0.10$? Why or why not?

$$P(\text{Type I}) = \alpha, \text{ so } P(\text{Type I}) = 0.10$$

No, if the null is true, and we fail to reject, this Type I error will occur 10% of the time just by chance since $P(\text{Type I}) = \alpha$, and $\alpha = .10$. The company should lower α .

(If students think Type II error more serious, increase α)