Notes #2
GEOMETRIC SEQUENCES

geometric sequence – a sequence of values that follow a pattern of multiplying each term by a fixed amount to arrive at the next term.

Common ratio – the fixed amount each term is multiplied by. To find the common ratio, divide the second term by the first term.

$(r = \frac{a_2}{a_1})$

Example 1: Is the given sequence geometric? If so, identify the common ratio.

A. 5, 15, 45, 135, ... $r = \frac{15}{5} = 3$

a_1 a_2

×3

yes $r = 3$

B. 15, 30, 45, 60, ... $r = \frac{30}{15} = 2$

×2

30 ×2 ≠ 45

no

Example 2: Find the common ratio of each sequence. Then find the next term.

A. 12, 18, 27, 40.5 ... 60.75

a_1 a_2

$r = \frac{a_2}{a_1} = \frac{18}{12} = \frac{3}{2}$

$r = \frac{3}{2}$ or 1.5

B. 98, 14, 2 ... $\frac{2}{7}$

$r = \frac{14}{98} = \frac{1}{7}$

80/100 = .80

80% = .80 a_1

Example 3: Suppose you drop a ball from a height of 100 cm. It bounces back to 80% of its previous height. How high will it go after its fifth bounce?

a_1 100 cm → a_2 80 → a_3 64 → a_4 51.2 → a_5 40.96

×.80 ×.80 ×.80 ×.80 40.96 cm

Example 4: Find the 4th term in each geometric sequence

a. $a_7 = 4$, $r = -\frac{1}{2}$

b. $a_2 = -9$, $r = \frac{1}{3}$

Geometric Sequence Formulas

Recursive Formula	*Explicit Formula*
a_1 = a given value	$a_n = a_1 (r)^{n-1}$
$a_n = a_{n-1} \cdot r$	$a_n = a_1 \cdot (r)^{n-1}$
Next = Now · Common Ratio	

In these formulas,

a_n is the nth term

a_1 is the first term

n is the number of the term

r is the common ratio

You can think of each term in the sequence as:

$a_1, a_2, a_3, a_4, \ldots, a_{n-1}, a_n, \ldots$

Example 5:

a. Write the recursive and explicit formulas for the geometric sequence with $a_1 = 6$ and $r = -2$.

R

$a_1 = 6$

$a_n = a_{n-1} \cdot -2$

E

$a_n = 6 \cdot (-2)^{n-1}$

b. Write the 1st ~~six~~ 4 terms of this geometric sequence.

a_1 6, a_2 −12, a_3 24, a_4 −48

×−2 ×−2 ×−2

c. Find the 50th term of the sequence.

$a_{50} = 6 \cdot (-2)^{50-1}$

$a_{50} = 6 \cdot (-2)^{49}$

$-3377,699,720,527,872$

Example 6: Use the sequences to find the recursive formula and explicit formula.

A. 66, 33, 16.5, 8.25.....

a_1 66

$r = \frac{33}{66} = \frac{1}{2}$

B. $33, 11, \frac{11}{3}, \frac{11}{9}, \ldots\ldots$

a) Recursive

$a_1 = 66$

$a_n = a_{n-1} \cdot \frac{1}{2}$

b) Explicit

$a_n = 66 \cdot \left(\frac{1}{2}\right)^{n-1}$

a) Recursive

b) Explicit